7.3.1:

$$\begin{aligned} x_1 \quad\quad - x_3 &= 0 \\ 3x_1 + x_2 + x_3 &= 1 \\ -x_1 + x_2 + 2x_3 &= 2 \end{aligned} \iff \underbrace{\begin{pmatrix} 1 & 0 & -1 \\ 3 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix}}_{\underline{\underline{A}}} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}_{\underline{x}} = \underbrace{\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}}_{\underline{b}}$$

$$|A| = \det(A) = 1(2-1) - 1(3-{}^{-}1) = 1-4 = -3$$

$$\Rightarrow \underline{x} = \underline{\underline{A}}^{-1}\underline{b}$$

$$A|I = \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 1 & 0 \\ -1 & 1 & 2 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 4 & -3 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 4 & -3 & 1 & 0 \\ 0 & 0 & 3 & 4 & -1 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 4 & -3 & 1 & 0 \\ 0 & 0 & 1 & -4/3 & 1/3 & -1/3 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/3 & 1/3 & -1/3 \\ 0 & 1 & 0 & 7/3 & -1/3 & 4/3 \\ 0 & 0 & 1 & -4/3 & 1/3 & -1/3 \end{array}\right)$$

where the right-hand block is A^{-1}.

$$A^{-1}b = \begin{pmatrix} -1/3 & 1/3 & -1/3 \\ 7/3 & -1/3 & 4/3 \\ -4/3 & 1/3 & -1/3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -1/3 \\ 7/3 \\ -1/3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

7.3.2:

$$\begin{aligned} x_1 + 2x_2 - x_3 &= 1 \\ 2x_1 + x_2 + x_3 &= 1 \\ x_1 - x_2 + 2x_3 &= 1 \end{aligned} \iff \underbrace{\begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & 1 \\ 1 & -1 & 2 \end{pmatrix}}_{\underline{A}} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}_{\underline{x}} = \underbrace{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}_{\underline{b}}$$

$$|A| = \det(A) = 1(2+1) - 2(4-1) - 1({}^{-}2-1) = 3-6+3 = 0$$

A is noninvertible

$$A|b = \left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 2 & 1 & 1 & 1 \\ 1 & -1 & 2 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -3 & 3 & -1 \\ 0 & -3 & 3 & 0 \end{array}\right)$$

$-3x_2 + 3x_3 = -1$ and
$-3x_2 + 3x_3 = 0$
cannot be solved together

7.3.3: $x_1 + 2x_2 - x_3 = 2$
$2x_1 + x_2 + x_3 = 1$
$x_1 - x_2 + 2x_3 = -1$
$\iff \underbrace{\begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & 1 \\ 1 & -1 & 2 \end{pmatrix}}_{\underline{\underline{A}}} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}_{\underline{x}} = \underbrace{\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}}_{\underline{b}}$

$|A| = \det(A) = 0$ from 7.3.2

$A|b = \left(\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 2 & 1 & 1 & 1 \\ 1 & -1 & 2 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -3 & 3 & -3 \\ 0 & -3 & 3 & -3 \end{array}\right)$

$-x_2 + x_3 = -1$
$x_3 = x_2 - 1$
$x_1 + 2x_2 - x_3 = 2$
$x_1 + 2x_2 - x_2 + 1 = 2$
$x_1 = 1 - x_2$

$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 - x_2 \\ x_2 \\ x_2 - 1 \end{pmatrix} = c_1 \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

or $x_1 = -c$, $x_2 = c+1$, $x_3 = c$ solves our equation

7.3.4: $x_1 + 2x_2 - x_3 = 0$
$2x_1 + x_2 + x_3 = 0$
$x_1 - x_2 + 2x_3 = 0$

$\det A = 0$ from 7.3.3

$(A|b) = \left(\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 2 & 1 & 1 & 0 \\ 1 & -1 & 2 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & -3 & 3 & 0 \end{array}\right)$

$-3x_2 + 3x_3 = 0$
$\Rightarrow x_2 = x_3$
$x_1 + 2x_2 - x_3 = 0$
$\Rightarrow x_1 + x_3 = 0$
$\Rightarrow x_1 = -x_3$

$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = c \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

7.3.5: $x_1 - x_3 = 0$
$3x_1 + x_2 + x_3 = 0$
$-x_1 + x_2 + 2x_3 = 0$
$\iff \underbrace{\begin{pmatrix} 1 & 0 & -1 \\ 3 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix}}_{A} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

From 7.3.1, $\det(A) = -3$, $x = A^{-1}b$, $A^{-1}b = 0 \Rightarrow$

$x = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$7.3.15: A = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} \quad |A-\lambda I| = \det \begin{pmatrix} 5-\lambda & -1 \\ 3 & 1-\lambda \end{pmatrix}$

$= (5-\lambda)(1-\lambda)+3 = 5-6\lambda+\lambda^2+3 = \lambda^2-6\lambda+8 = (\lambda-4)(\lambda-2)$

$\Rightarrow \lambda_1 = 2, \ \lambda_2 = 4$

$\lambda_1 = 2: \ (A-\lambda_1 I)u = 0 \Rightarrow \begin{pmatrix} 3 & -1 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad u_1 = \frac{u_2}{3}$

$u_1 = \binom{1}{3}$

$\lambda_2 = 4: \ (A-\lambda_2 I)v = 0 \Rightarrow \begin{pmatrix} 1 & -1 \\ 3 & -3 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad v_1 = v_2$

$v = \binom{1}{1}$

$7.3.16: A = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} \quad |A-\lambda I| = \det \begin{pmatrix} 3-\lambda & -2 \\ 4 & -1-\lambda \end{pmatrix}$

$= (3-\lambda)(-1-\lambda)+8 = \lambda^2-2\lambda+5 = 0 \Rightarrow \lambda = \frac{2 \pm \sqrt{4-20}}{2} = 1 \pm 2i$

$\lambda_1 = 1+2i \quad (A-\lambda_1 I)u = 0 \Rightarrow \begin{pmatrix} 2-2i & -2 \\ 4 & -2-2i \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow (1-i)u_1 = u_2$

$u = \binom{1}{1-i}$

$\lambda_2 = 1-2i \quad (A-\lambda_2 I)\cancel{u}v = 0 \Rightarrow \begin{pmatrix} 2+2i & -2 \\ 4 & -2+2i \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow (1+i)v_1 = v_2$

$v = \binom{1}{1+i}$

$7.3.17: A = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \quad |A-\lambda I| = \det \begin{pmatrix} -2-\lambda & 1 \\ 1 & -2-\lambda \end{pmatrix}$

$= (-2-\lambda)(-2-\lambda)-1 = \lambda^2+4\lambda+3 = (\lambda+3)(\lambda+1)$

$\Rightarrow \lambda_1 = -1, \ \lambda_2 = -3$

$\lambda_1 = -1: \ (A-\lambda_1 I)u = 0 \Rightarrow \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow u_1 = u_2$

$u = \binom{1}{1}$

$\lambda_2 = -3: \ (A-\lambda_2 I)v = 0 \Rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow v_1 = -v_2$

$v = \binom{1}{-1}$

$7.3.18: A = \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \quad |A-\lambda I| = \det \begin{pmatrix} 1-\lambda & i \\ -i & 1-\lambda \end{pmatrix} = (1-\lambda)^2 - 1 = \lambda^2 - 2\lambda$

$\Rightarrow \lambda_1 = 0, \ \lambda_2 = 2$

$\lambda_1 = 0: \ (A-\lambda_1 I)u = 0 \Rightarrow \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow iu_2 = -u_1$

$u = \binom{1}{i}$

$\lambda_2 = 2: \ (A-\lambda_2 I)v = 0 \Rightarrow \begin{pmatrix} -1 & i \\ -i & -1 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow v_1 = iv_2$

$v = \binom{1}{-i}$

7.3.19: $A = \begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$ $|A - \lambda I| = \det\begin{pmatrix} 1-\lambda & \sqrt{3} \\ \sqrt{3} & -1-\lambda \end{pmatrix} = 0$

$= (1-\lambda)(-1-\lambda) - 3 = \lambda^2 - 4 = (\lambda-2)(\lambda+2) \Rightarrow \lambda_1 = 2, \lambda_2 = -2$

$\lambda_1 = 2: (A - \lambda_1 I)\underline{u} = 0 \Rightarrow \begin{pmatrix} -1 & \sqrt{3} \\ \sqrt{3} & -3 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow u_1 = \sqrt{3}\, u_2$

$\underline{u} = \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix}$

$\lambda_2 = -2: (A - \lambda_2 I)\underline{v} = 0 \Rightarrow \begin{pmatrix} 3 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \sqrt{3}\, v_1 = -v_2$

$\underline{v} = \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix}$

7.3.20: $A = \begin{pmatrix} -3 & 3/4 \\ -5 & 1 \end{pmatrix}$ $|A - \lambda I| = \det\begin{pmatrix} -3-\lambda & 3/4 \\ -5 & 1-\lambda \end{pmatrix}$

$= (-3-\lambda)(1-\lambda) + 15/4 = \lambda^2 + 2\lambda + 3/4 = (\lambda + 3/2)(\lambda + 1/2) = 0$

$\lambda_1 = -3/2: (A - \lambda_1 I)\underline{u} = 0 \Rightarrow \begin{pmatrix} -3/2 & 3/4 \\ -5 & 5/2 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \underline{u} = \begin{pmatrix} 1/2 \\ 1 \end{pmatrix}$

$\lambda_2 = -1/2: (A - \lambda_2 I)\underline{v} = 0 \Rightarrow \begin{pmatrix} -5/2 & 3/4 \\ -5 & 3/2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \underline{v} = \begin{pmatrix} 3 \\ 10 \end{pmatrix}$

7.3.21: $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{pmatrix}$ $|A - \lambda I| = \det\begin{pmatrix} 1-\lambda & 0 & 0 \\ 2 & 1-\lambda & -2 \\ 3 & 2 & 1-\lambda \end{pmatrix} = 0$

$= (1-\lambda)[(1-\lambda)^2 + 4] = (1-\lambda)[\lambda^2 - 2\lambda + 5] \Rightarrow \lambda = 1, 1 \pm 2i$

$\lambda_1 = 1;\ \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & -2 \\ 3 & 2 & 0 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \underline{u} = \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix}$

$\lambda_2 = 1 + 2i:\ \begin{pmatrix} -2i & 0 & 0 \\ 2 & -2i & -2 \\ 3 & 2 & -2i \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \underline{v} = \begin{pmatrix} 0 \\ i \\ 1 \end{pmatrix}$

$\lambda_3 = 1 - 2i;\ \begin{pmatrix} 2i & 0 & 0 \\ 2 & 2i & -2 \\ 3 & 2 & 2i \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \underline{w} = \begin{pmatrix} 0 \\ -i \\ 1 \end{pmatrix}$

7.3.22: $A = \begin{pmatrix} 3 & 2 & 2 \\ 1 & 4 & 1 \\ -2 & -4 & -1 \end{pmatrix}$ $|A - \lambda I| = \det\begin{pmatrix} 3-\lambda & 2 & 2 \\ 1 & 4-\lambda & 1 \\ -2 & -4 & -1-\lambda \end{pmatrix}$

$= (3-\lambda)[\lambda^2 - 3\lambda - 4 + 4] - 2[-1-\lambda+2] + 2[-4 + 8 - 2\lambda]$

$= (3-\lambda)(\lambda^2 - 3\lambda) + 2(\lambda - 1 - 2\lambda + 4) = (3-\lambda)(\lambda^2 - 3\lambda + 2)$

$= (3-\lambda)(\lambda-2)(\lambda-1) = 0$

$\Rightarrow \lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3$

7.3.22:

$\lambda_1 = 1:\ \begin{pmatrix} 2 & 2 & 2 \\ 1 & 3 & 1 \\ -2 & -4 & -2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \underline{u} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

$\lambda_2 = 2:\ \begin{pmatrix} 1 & 2 & 2 \\ 1 & 2 & 1 \\ -2 & -4 & -3 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \underline{v} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$

$\lambda_3 = 3:\ \begin{pmatrix} 0 & 2 & 2 \\ 1 & 1 & 1 \\ -2 & -4 & -4 \end{pmatrix}\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \underline{\omega} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

7.3.23: $A = \begin{pmatrix} 11/9 & -2/9 & 8/9 \\ -2/9 & 2/9 & 10/9 \\ 8/9 & 10/9 & 5/9 \end{pmatrix}$ $|A - \lambda I| = \det\begin{pmatrix} 11/9-\lambda & -2/9 & 8/9 \\ -2/9 & 2/9-\lambda & 10/9 \\ 8/9 & 10/9 & 5/9-\lambda \end{pmatrix}$

$= (11/9 - \lambda)\left[\lambda^2 - 7/9\,\lambda + 10/81 - \frac{100}{81}\right] + 2/9\left(\frac{-10}{81} + \frac{2}{9}\lambda - \frac{80}{81}\right)$
$+ 8/9\left(-\frac{20}{81} - \frac{16}{81} + \frac{8}{9}\lambda\right) = \left(\frac{11}{9} - \lambda\right)\left(\lambda^2 - 7/9\,\lambda - \frac{10}{9}\right) + \frac{2}{9}\left(\frac{2}{9}\lambda - \frac{10}{9}\right)$
$+ \frac{8}{9}\left(\frac{8}{9}\lambda - \frac{4}{9}\right) = 11/9\,\lambda^2 - \frac{77}{81}\lambda - \frac{110}{81} - \lambda^3 + 7/9\,\lambda^2 + \frac{10}{9}\lambda + \frac{4}{81}\lambda$
$- \frac{20}{81} + \frac{64}{81}\lambda - \frac{32}{81} = -\lambda^3 + 2\lambda^2 + \lambda - 2 = (\lambda - 2)(-\lambda^2 + 1) = 0$
$\Rightarrow \lambda_1 = 1,\ \lambda_2 = -1,\ \lambda_3 = 2$

$\lambda_1 = 1:\ \begin{pmatrix} 2/9 & -2/9 & 8/9 \\ -2/9 & -7/9 & 10/9 \\ 8/9 & 10/9 & -4/9 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \underline{u} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$

$\lambda_2 = -1:\ \begin{pmatrix} 20/9 & -2/9 & 8/9 \\ -2/9 & 11/9 & 10/9 \\ 8/9 & 10/9 & 14/9 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \underline{v} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$

$\lambda_3 = 2:\ \begin{pmatrix} -7/9 & -2/9 & 8/9 \\ -2/9 & -16/9 & 10/9 \\ 8/9 & 10/9 & -13/9 \end{pmatrix}\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \underline{\omega} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$

$$7.3.24: A = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix} \quad |A - \lambda I| = \det\begin{pmatrix} 3-\lambda & 2 & 4 \\ 2 & -\lambda & 2 \\ 4 & 2 & 3-\lambda \end{pmatrix}$$

$$= (3-\lambda)(\lambda^2 - 3\lambda - 4) - 2(6 - 2\lambda - 8) + 4(4 + 4\lambda)$$

$$= (3-\lambda)(\lambda - 4)(\lambda+1) + 4(\lambda+1) + 16(\lambda+1)$$

$$= (\lambda+1)(-\lambda^2 + 7\lambda - 12 + 4 + 16) = (\lambda+1)(-\lambda^2 + 7\lambda + 8)$$

$$= -(\lambda+1)(\lambda-8)(\lambda+1) = 0 \Rightarrow \lambda_1 = -1, \lambda_2 = -1, \lambda_3 = 8$$

$$\lambda_1 = -1: \quad \begin{pmatrix} 4 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 4 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \underline{u} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$\lambda_2 = -1: \quad \begin{pmatrix} 4 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 4 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \underline{v} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$$

$$\lambda_3 = 8: \quad \begin{pmatrix} -5 & 2 & 4 \\ 2 & -8 & 2 \\ 4 & 2 & -5 \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \underline{w} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$