7.1.1: $u'' + .5u' + 2u = 0$ $\qquad u' = v$

$v' = u''$

$\Rightarrow u' = v$

$v' = -.5v - 2u$

7.1.2: $u'' + .5u' + 2u = 3\sin t$ $\qquad u' = v,\ v' = u''$

$\Rightarrow u' = v$

$v' = -2u - .5v + 3\sin t$

7.1.3: $t^2 u'' + tu' + (t^2 - .25)u = 0$ $\qquad u' = v,\ v' = u''$

$\Rightarrow u' = v$

$v' = -(1 - \frac{.25}{t^2})u - \frac{1}{t}v$

7.1.4: $u^{(4)} - u = 0 \Rightarrow$ $\quad u' = v$

$v' = w$

$w' = x$

$x' = u$

7.1.7: $x_1' = -2x_1 + x_2$

$x_2' = x_1 - 2x_2$

a) $x_2 = 2x_1 + x_1'$

$x_2' = 2x_1' + x_1'' = -2(2x_1 + x_1') + x_1$

$\Rightarrow x_1'' + 4x_1' + 3x_1 = 0$

$x_1 = ce^{rt} \Rightarrow \quad ce^{rt}(r^2 + 4r + 3) = 0$

$r = -3, -1$

$\Rightarrow x_1 = c_1 e^{-3t} + c_2 e^{-t}$

$x_2 = 2x_1 + x_1' = 2c_1 e^{-3t} + 2c_2 e^{-t} - 3c_1 e^{-3t} - c_2 e^{-t}$

$= -c_1 e^{-3t} + c_2 e^{-t}$

7.1.7b: $x_1(0) = 2 = c_1 \cdot 1 + c_2 \cdot 1$

$$x_2(0) = 3 = -c_1 \cdot 1 + c_2 \cdot 1$$

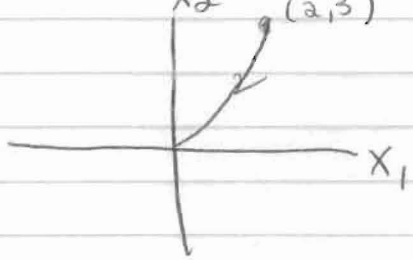
$$5 = 0 + 2c_2 \Rightarrow c_2 = 5/2$$

$$\Rightarrow c_1 = -1/2$$

$$x_1(t) = -\tfrac{1}{2} e^{-3t} + \tfrac{5}{2} e^{-t}$$

$$x_2(t) = \tfrac{1}{2} e^{-3t} + \tfrac{5}{2} e^{-t}$$

7.1.7.c:



7.1.8: $x_1' = 3x_1 - 2x_2$ $\quad x_1(0) = 3$

$$x_2' = 2x_1 - 2x_2 \qquad x_2(0) = 1/2$$

$$x_2 = 3/2\, x_1 - 1/2\, x_1'$$

$$\Rightarrow \tfrac{3}{2} x_1' - \tfrac{1}{2} x_1'' = 2x_1 - 3x_1 + x_1'$$

$$\Rightarrow 0 = x_1'' - x_1' - 2x_1 \qquad x_1 = ce^{rt}$$

$$0 = r^2 - r - 2 = (r-2)(r+1)\ ;\ r = 2, -1$$

$$x_1 = c_1 e^{2t} + c_2 e^{-t}$$

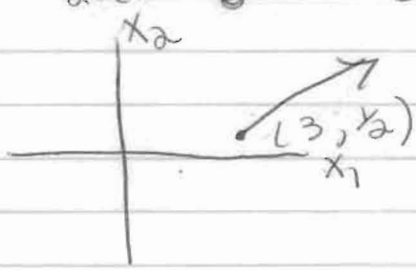
$$x_2 = \tfrac{3}{2} c_1 e^{2t} + \tfrac{3}{2} c_2 e^{-t} - c_1 e^{2t} + \tfrac{1}{2} c_2 e^{-t} = \tfrac{1}{2} c_1 e^{2t} + 2c_2 e^{-t}$$

$$3 = c_1 + c_2$$

$$\tfrac{1}{2} = \tfrac{1}{2} c_1 + 2c_2 \Rightarrow 2 = -3c_2 \Rightarrow c_2 = -2/3,\ c_1 = 11/3$$

$$x_1(t) = 11/3\, e^{2t} - 2/3\, e^{-t}$$

$$x_2(t) = \tfrac{11}{6} e^{2t} - 4/3\, e^{-t}$$



7.1.9: $x_1' = \frac{5}{4}x_1 + \frac{3}{4}x_2$ $\quad x_1(0) = -2$

$x_2' = \frac{3}{4}x_1 + \frac{5}{4}x_2$ $\quad x_2(0) = 1$

$x_2 = \frac{4}{3}x_1' - \frac{5}{3}x_1$

$\Rightarrow \frac{4}{3}x_1'' - \frac{5}{3}x_1' = \frac{3}{4}x_1 + \frac{5}{3}x_1' - \frac{25}{12}x_1$

$\Rightarrow x_1'' - \frac{5}{2}x_1' + x_1 = 0 \Rightarrow x = ce^{rt}, \quad r^2 - \frac{5}{2}r + 1 = 0$

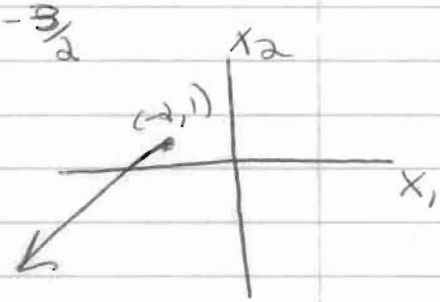
$\Rightarrow (r-2)(r-\frac{1}{2}) = 0 \quad r = 2, \frac{1}{2}$

$x_1(t) = c_1e^{2t} + c_2e^{\frac{1}{2}t}$; $x_2(t) = \frac{8}{3}c_1e^{2t} + \frac{2}{3}c_2e^{\frac{1}{2}t} - \frac{5}{3}c_1e^{2t} - \frac{5}{3}c_2e^{\frac{1}{2}t}$

$x_2(t) = c_1e^{2t} - c_2e^{\frac{1}{2}t}$

$-2 = c_1 + c_2 \Rightarrow c_1 = -\frac{1}{2}, c_2 = -\frac{3}{2}$

$1 = c_1 - c_2$



$x_1(t) = \frac{-1}{2}e^{2t} - \frac{3}{2}e^{\frac{1}{2}t}$

$x_2(t) = -\frac{1}{2}e^{2t} + \frac{3}{2}e^{\frac{1}{2}t}$

7.1.10: $x_1' = x_1 - 2x_2$ $\quad x_1(0) = -1$

$x_2' = 3x_1 - 4x_2$ $\quad x_2(0) = 2$

$x_2 = \frac{1}{2}x_1 - \frac{1}{2}x_1'$

$\Rightarrow \frac{1}{2}x_1' - \frac{1}{2}x_1'' = 3x_1 - 2x_1 + 2x_1'$

$\Rightarrow 0 = \frac{1}{2}x_1'' + \frac{3}{2}x_1' + x_1 \Rightarrow 0 = x_1'' + 3x_1' + 2x_1$

$r^2 + 3r + 2 = 0 \Rightarrow (r+2)(r+1) = 0, \quad x = ce^{rt}$

$x_1(t) = c_1e^{-t} + c_2e^{-2t}$; $x_2(t) = \frac{c_1}{2}e^{-t} + \frac{c_2}{2}e^{-2t} + \frac{c_1}{2}e^{-t} + c_2e^{-2t}$

$x_2(t) = c_1e^{-t} + \frac{3}{2}c_2e^{-2t}$

$-1 = c_1 + c_2 \Rightarrow c_2 = 6, c_1 = -7$

$2 = c_1 + \frac{3}{2}c_2$

$x_1(t) = -7e^{-t} + 6e^{-2t}$

$x_2(t) = -7e^{-t} + 9e^{-2t}$

7.1.11: $x_1' = 2x_2 \Rightarrow \frac{1}{2}x_1'' = -2x_1 \Rightarrow x_1'' + 4x_1 = 0$

$x_2' = -2x_1 \qquad x_1 = ce^{rt} \Rightarrow r^2 + 4 = 0$

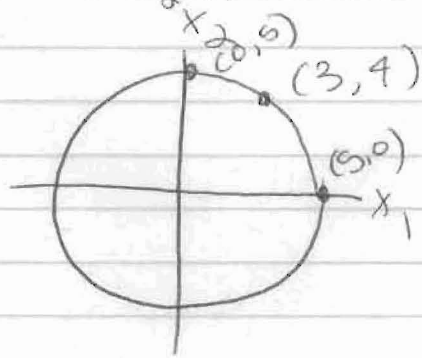
$r = \pm 2i$

$x_1(t) = c_1\cos 2t + c_2 \sin 2t \qquad e^{2i} = \cos 2t + i\sin 2t$

$x_2(t) = -1 \cdot c_1 \sin t + c_2 \cos t$

$x_1(0) = 3 = c_1 \qquad x_1(t) = 3\cos 2t + 4\sin 2t$

$x_2(0) = 4 = c_2 \qquad x_2(t) = 4\cos 2t - 3\sin 2t$



7.1.12: $x_1' = -.5x_1 + 2x_2 \qquad x_1(0) = -2$

$x_2' = -2x_1 - .5x_2 \qquad x_2(0) = 2$

$x_2 = \frac{1}{2}x_1' + \frac{1}{4}x_1$

$\Rightarrow \frac{1}{2}x_1'' + \frac{1}{4}x_1' = -2x_1 - \frac{1}{4}x_1' - \frac{1}{8}x_1$

$\Rightarrow 4x_1'' + 2x_1' = -16x_1 - 2x_1' - x_1$

$\Rightarrow 4x_1'' + 4x_1' + 17x_1 = 0$

$\Rightarrow 4r^2 + 4r + 17 = 0 \Rightarrow r = \frac{-4 \pm \sqrt{16 - 4(4)(17)}}{2 \cdot 4}$

$\Rightarrow r = \frac{-4 \pm \sqrt{-256}}{8} = -\frac{1}{2} \pm 2i \qquad , x = ce^{rt}$

$x_1 = c_1 e^{-\frac{1}{2}t}\cos(2t) + c_2 e^{-\frac{1}{2}t}\sin(2t)$

$x_2 = \frac{c_1}{2}\cdot(-\frac{1}{2})e^{-\frac{1}{2}t}\cos(2t) + \frac{c_1}{2}e^{-\frac{1}{2}t}(2 \cdot -\sin(2t))$

$+ \frac{c_2}{2}\cdot(-\frac{1}{2})e^{-\frac{1}{2}t}\sin(2t) + \frac{c_2}{2}e^{-\frac{1}{2}t}(2\cos 2t)$

$+ \frac{c_1}{4}e^{-\frac{1}{2}t}\cos(2t) + \frac{c_2}{4}e^{-\frac{1}{2}t}\sin(2t)$

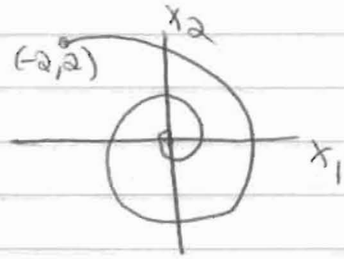
$= e^{-\frac{1}{2}t}\cos(2t)\left(-\frac{c_1}{4} + c_2 + \frac{c_1}{4}\right) + e^{-\frac{1}{2}t}\sin(2t)\left(-c_1 - \frac{c_2}{4} + \frac{c_2}{4}\right)$

$= c_2 e^{-\frac{1}{2}t}\cos(2t) - c_1 e^{-\frac{1}{2}t}\sin(2t)$

$-2 = c_1 \Rightarrow x_1(t) = -2e^{-\frac{1}{2}t}\cos(2t) + 2e^{-\frac{1}{2}t}\sin(2t)$

$2 = c_2 \qquad x_2(t) = 2e^{-\frac{1}{2}t}\cos(2t) + 2e^{-\frac{1}{2}t}\sin(2t)$

7.1.12:



7.1.22: a) $Q_1'(t) = 1 \cdot 1.5 - \frac{3}{30} Q_1 + \frac{1.5}{20} Q_2 \quad Q_1(0) = 25$

$Q_2'(t) = 3 \cdot 1 + \frac{3}{30} Q_1 - \frac{4}{20} Q_2 \quad Q_2(0) = 15$

$\Rightarrow Q_1'(t) = \frac{3}{2} - \frac{1}{10} Q_1 + \frac{3}{40} Q_2$

$Q_2'(t) = 3 + \frac{1}{10} Q_1 - \frac{1}{5} Q_2$

b) $Q_1'(t) = 0 = 3/2 - \frac{1}{10} Q_1^E + 3/40 Q_2^E$

$Q_2'(t) = 0 = 3 + \frac{1}{10} Q_1^E - \frac{1}{5} Q_2^E$

$\Rightarrow 0 = 9/2 - 5/40 Q_2^E \Rightarrow Q_2^E = \frac{9}{2} \cdot \frac{40}{5} = 36$

$0 = 3/2 - \frac{1}{10} Q_1^E + \frac{3}{40} \cdot 36 \Rightarrow \frac{1}{10} Q_1^E = \frac{3}{2} + \frac{27}{10}$

$\Rightarrow Q_1^E = 42$, Q_2 will be faster

c) $x_1 = Q_1(t) - Q_1^E \Rightarrow x_1' = Q_1'(t).$

$x_2 = Q_2(t) - Q_2^E \qquad x_2' = Q_2'(t)$

$x_1' = \frac{3}{2} - \frac{1}{10}(x_1 + Q_1^E) + \frac{3}{40}(x_2 + Q_2^E) = \frac{3}{2} - \frac{1}{10} x_1 - \frac{42}{10} + \frac{3}{40} x_2 + \frac{27}{10}$

$= \frac{15 + 27 - 42}{10} - \frac{1}{10} x_1 + \frac{3}{40} x_2 = -\frac{1}{10} x_1 + \frac{3}{40} x_2$

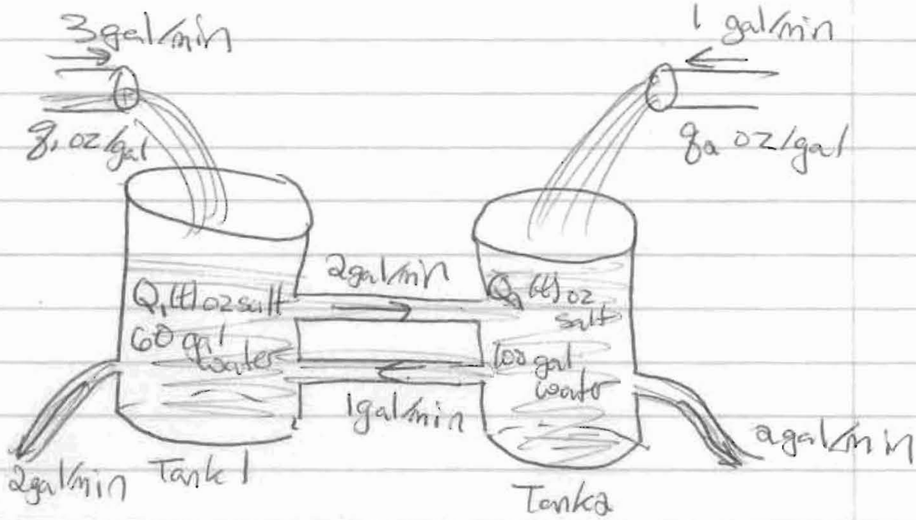
$x_2' = 3 + \frac{1}{10}(x_1 + Q_1^E) - \frac{1}{5}(x_2 + Q_2^E) = 3 + \frac{1}{10} x_1 + \frac{42}{10} - \frac{1}{5} x_2 - \frac{36}{5}$

$= \frac{30 + 42 - 72}{10} + \frac{1}{10} x_1 - \frac{1}{5} x_2 = \frac{1}{10} x_1 - \frac{1}{5} x_2$

$x_1(0) = Q_1(0) - Q_1^E = 25 - 42 = -17$

$x_2(0) = Q_2(0) - Q_2^E = 15 - 36 = -21$

7.1.23:



$Q_1'(t) = 3q_1 + \frac{1}{100}Q_2 - \frac{2+2}{60}Q_1 = 3q_1 + \frac{Q_2}{100} - \frac{Q_1}{15}$

$Q_2'(t) = q_2 + \frac{2}{60}Q_1 - \frac{1+2}{100}Q_2 = q_2 + \frac{Q_1}{30} - \frac{3}{100}Q_2$

$Q_1(0) = Q_1^0$, $Q_2(0) = Q_2^0$

b) $0 = 3q_1 + \frac{1}{100}Q_2^E - \frac{1}{15}Q_1^E \Rightarrow 9q_1 + q_2 - \frac{5}{30}Q_1^E = 0$

$0 = q_2 - \frac{3}{100}Q_2^E + \frac{1}{30}Q_1^E$

$\Rightarrow Q_1^E = 54q_1 + 6q_2$

$0 = q_2 - \frac{3}{100}Q_2^E + \frac{54}{30}q_1 + \frac{6}{30}q_2$

$\Rightarrow Q_2^E = \frac{100}{3}\left(\frac{36}{30}q_2 + \frac{54}{30}q_1\right) = 60q_1 + 40q_2$

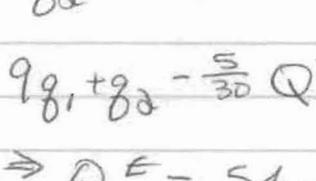
c) Only if $60q_1 + 40q_2 = 50$

$54q_1 + 6q_2 = 60$

$\begin{bmatrix} 60 & 40 \\ 54 & 6 \end{bmatrix} = A$ $\det(A) = 360 - 2160 \neq 0$

$A^{-1} = \begin{bmatrix} -\frac{1}{300} & \frac{1}{45} \\ \frac{3}{100} & -\frac{1}{30} \end{bmatrix}$ $A^{-1}b = \begin{bmatrix} -\frac{1}{300} & \frac{1}{45} \\ \frac{3}{100} & -\frac{1}{30} \end{bmatrix}\begin{bmatrix} 50 \\ 60 \end{bmatrix} = \begin{bmatrix} \frac{7}{6} \\ -\frac{1}{2} \end{bmatrix}$

q_2 cannot be negative, so, although the equation can be solved, the solution is not valid.



d) (1) $q_2 = \frac{Q_1^E}{6} - 9q_1$

(2) $q_2 = \frac{Q_2^E}{40} - \frac{3}{2}q_1$

The y intercept of eq (1) is $\frac{Q_1^E}{6}$ and eq (2) is $Q_2^E/40$. The x intercepts are $\frac{Q_1^E}{54}$ and $\frac{Q_2^E}{60}$.

There is a unique valid soln as long as $\frac{Q_1^E}{54} \le \frac{Q_2^E}{60}$ or $\frac{Q_2^E}{40} \le \frac{Q_1^E}{6}$, i.e. $10/9 \le \frac{Q_2}{Q_1} \le \frac{20}{3}$