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MR1313756 (95k:15037) 15A60 (47A12)

Johnson, Charles R. [Johnson, Charles Royal] (1-CWM); **Spitkovsky, I. M.** (1-CWM); **Gottlieb, S. [Gottlieb, Sigal]** (1-BRN-A)

Inequalities involving the numerical radius. (English summary)

Special Issue: The numerical range and numerical radius.

Linear and Multilinear Algebra **37** (1994), *no. 1-3,* 13–24.

Let A, B be $n \times n$ complex matrices. $F(A)$ denotes the classical numerical range, $r(A) = \max\{|z|\colon z \in F(A)\}$ is the numerical radius of A. The power inequality $r(A^m) \leq (r(A))^m$ is well known but the inequality $r(AB) \leq r(A)r(B)$ is not generally true [see P. R. Halmos, *A. Hilbert space problem book*, Van Nostrand, Princeton, NJ, 1967; MR0208368 (34 #8178)]. The authors consider the interesting question for which polynomials f and which matrices A does the inequality $(*)$ $r(f(A)) \leq f(r(A))$ hold. In general this seems to be a difficult problem. A few necessary and some sufficient conditions are given in the case $n = 2$. The authors obtain, for example, the following results: If f satisfies $(*)$ for all A, then $f'(x) \geq 0$ for $x > 0$. If $f(x) > 0$, $f'(x) > 0$ for $x > 0$, then $(*)$ is true for all 2×2 matrices A with real nonnegative eigenvalues. Moreover, a result for the inequality $r(f(A)g(A)) \leq r(f(A))r(g(A))$ is given.

Reviewed by *Adolf Rhodius*

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