Applied Math 9

Problem Set 3 for Zero Sum Games

1. For each of the following systems of equations, use the determinant to decide if there is a unique solution.

$$(2,1)\cdot(x_1,x_2)=3, (4,2)\cdot(x_1,x_2)=3$$

$$(1,4)\cdot(x_1,x_2)=2, (-1,4)\cdot(x_1,x_2)=3$$

$$(2,1,0)\cdot(x_1,x_2,x_3)=1, (1,1,1)\cdot(x_1,x_2,x_3)=-2, (3,2,1)\cdot(x_1,x_2,x_3)=0$$

$$(1,1,0)\cdot(x_1,x_2,x_3)=1, (0,1,0)\cdot(x_1,x_2,x_3)=5, (1,1,1)\cdot(x_1,x_2,x_3)=-1$$

2. A set of points S in n-dimensional space is called *convex* if given any two points $(x_1,x_2,\dots,x_n)$ and $(y_1,y_2,\dots,y_n)$ in S, the line joining these points also lies in S. In other words, for any $0\le t\le 1$,

$$(x_1,x_2,\dots,x_n)t+(y_1,y_2,\dots,y_n)(1-t)\in S.$$

Are half-spaces convex? Is the intersection of 2 or more half spaces convex?

3. In class we discussed how to turn the Holmes-Moriarty lower game with mixed strategies into a maximization problem. Show that the upper can also be turned into a maximization problem.

4. A function f on n-dimensional space is called *affine* if it is the sum of a dot product and a constant. In other words, for some known vector $(a_1,a_2,\dots,a_n)$ and number b and with variable $(x_1,x_2,\dots,x_n)$,

$$f(x_1,x_2,\dots,x_n)=(a_1,a_2,\dots,a_n)\cdot(x_1,x_2,\dots,x_n)+b.$$

Suppose that S is convex. A point $(x_1,x_2,\dots,x_n)\in S$ is called a *non-extreme* point if there is a direction $(v_1,v_2,\dots,v_n)$, such that small movements back and forth in the direction $(v_1,v_2,\dots,v_n)$ around $(x_1,x_2,\dots,x_n)$ are still in S. (In precise mathematical terms, we would say that there is $c>0$ such that

$$(x_1,x_2,\dots,x_n)+t(v_1,v_2,\dots,v_n)\in S$$

as long as $|t|<c$.) All other points are called *extreme* points. Do you think an affine function can take on its maximum value at a non-extreme point and be strictly smaller at all other points in S?

5. Let A be an $n \times n$ matrix with determinant 0 and let B be an other $n \times n$ matrix. What are $\det(AB)$ and $\det(BA)$? (It may be useful to think of A and B as defining transformations and use the volume interpretation of determinant.)