AM 034 — Applied Mathematics - II

Brown University | **Spring 2022**

Homework, Set 5 | **Due March 16**

For each of the following matrices,

$$\begin{bmatrix} -2 & 1 \\ 6 & 3 \end{bmatrix}, \quad \begin{bmatrix} -3 & -4 \\ 5 & 1 \end{bmatrix}, \quad \begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix}, \quad \begin{bmatrix} -5 & -2 \\ 8 & 3 \end{bmatrix}, \quad \begin{bmatrix} -6 & -8 \\ 5 & 6 \end{bmatrix}, \quad \begin{bmatrix} -2 & -4 \\ 4 & 6 \end{bmatrix}, \quad \begin{bmatrix} 7 & 1 \\ 6 & 6 \end{bmatrix},$$

consider the constant coefficient vector differential equation

$$\dot{\mathbf{y}}(t) = \mathbf{A}\,\mathbf{y}(t).$$

5.1 (30 points) Determine the **type** of critical point at the origin

5.2 (30 points) Determine the **stability** of the critical point at the origin.

5.3 (40 points) **Plot** a phase portrait (direction field along with some solutions) to confirm your answers in two previous parts. If it is a node or saddle point, add the graph of the corresponding separatrix to your plot.
Please, send your **code** as attachement in a separate file—I don't need your pictures.

You may find the following table useful:

Eigenvalues	**Type of Critical Point**	**Stability**
$\lambda_1 > \lambda_2 > 0$	Nodal source (node)	Unstable
$\lambda_1 < \lambda_2 < 0$	Nodal sink (node)	Asymptotically stable
$\lambda_1 < 0 < \lambda_2$	Saddle point	Unstable
$\lambda_1 = \lambda_2 > 0$, diagonal matrix	Proper node/star point	Unstable
$\lambda_1 = \lambda_2 < 0$, diagonal matrix	Proper node/star point	Asymptotically stable
$\lambda_1 = \lambda_2 > 0$, missing eigenvector	Improper/degenerate node	Unstable
$\lambda_1 = \lambda_2 < 0$, missing eigenvector	Improper/degenerate node	Asymptotically stable
$\lambda = \alpha \pm \mathbf{j}\beta$, $\alpha > 0$	Spiral point	Unstable
$\lambda = \alpha \pm \mathbf{j}\beta$, $\alpha < 0$	Spiral point	Asymptotically stable
$\lambda = \pm\beta\mathbf{j}$	Center	Stable