AM 034 — Applied Mathematics - II

Brown University | **Spring 2022**

Homework, Set 4 | **Due March 09**

For each of the following matrices,

$$\mathbf{A} = \begin{bmatrix} -67 & -49 & 87 \\ -52 & -34 & 69 \\ -88 & -62 & 115 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} -55 & -40 & 72 \\ -112 & -79 & 144 \\ -112 & -80 & 145 \end{bmatrix},$$

4.1 (10 points) find at least two square roots

4.2 (10 points) determine the exponential matrix $\mathbf{U}(t) = e^{\mathbf{A}\,t}$, $\quad \mathbf{U}(t) = e^{\mathbf{B}\,t}$,

4.3 (20 points) show that this matrix function is a solution of the initial value problem

$$\dot{\mathbf{U}}(t) = \mathbf{A}\,\mathbf{U}(t), \qquad \mathbf{U}(0) = \mathbf{I}$$

and

$$\dot{\mathbf{U}}(t) = \mathbf{B}\,\mathbf{U}(t), \qquad \mathbf{U}(0) = \mathbf{I},$$

respectively;

4.4 (10 points) find the matrix-function $\mathbf{\Phi}(t) = \dfrac{\sin\left(\sqrt{\mathbf{A}}\,t\right)}{\sqrt{\mathbf{A}}}$, $\quad \mathbf{\Phi}(t) = \dfrac{\sin\left(\sqrt{\mathbf{B}}\,t\right)}{\sqrt{\mathbf{B}}}$;

4.5 (20 points) show that the matrix function is a solution of the following initial avlue problem

$$\ddot{\mathbf{\Phi}}(t) + \mathbf{A}\,\mathbf{\Phi}(t) = \mathbf{0}, \qquad \mathbf{\Phi}(0) = \mathbf{0}, \quad \dot{\mathbf{\Phi}}(0) = \mathbf{I}$$

and

$$\ddot{\mathbf{\Phi}}(t) + \mathbf{B}\,\mathbf{\Phi}(t) = \mathbf{0}, \qquad \mathbf{\Phi}(0) = \mathbf{0}, \quad \dot{\mathbf{\Phi}}(0) = \mathbf{I}$$

respectively;

4.6 (10 points) construct the matrix function $\mathbf{\Psi}(t) = \cos\left(\sqrt{\mathbf{A}}\,t\right)$, $\quad \mathbf{\Psi}(t) = \cos\left(\sqrt{\mathbf{B}}\,t\right)$

4.7 (20 points) show that they satisfy the same differential equation as in 4.5, but subject to the initial conditions

$$\mathbf{\Psi}(0) = \mathbf{I}, \quad \dot{\mathbf{\Psi}}(0) = \mathbf{0}.$$