# AM 034 — Applied Mathematics - II

**Brown University** **Spring 2022**
**Homework, Set 1** **Due February 09**

**1.1** (30 points) Suppose that a projectile is launched into the air from ground with initial velocity $v_0$ and angle $\alpha$ with the horizontal plane. Assuming that there is no air resistance, the model for its movement is as follows

$$\begin{aligned} \ddot{x} &= 0, \qquad & x(0) = 0, \quad \dot{x}(0) = v_0 \cos\alpha, \\ \ddot{y} &= -g; \qquad & y(0) = 0, \quad \dot{y}(0) = v_0 \sin\alpha. \end{aligned}$$

Here $g \approx 9.81$ is the acceleration due to gravity and overdot denotes the derivative with respect to time $t$.

**(a)** (5 points) Show that the projectile's trajectory is a parabola.

**(b)** (5 points) Confirm part (a) by plotting several trajectories on an $x - y$ plane.

**(c)** (10 points) Let $(x_m, y_m)$ be coordinates of the vertex corresponding to the parabola solution curve, so $y_m$ is the maximum height attained and $x_m$ is the corresponding horizontal coordinate. Plot the curve with coordinates $(x_m, y_m)$ for different launch angles $\alpha$.

**(d)** (10 points) Find the formula for the range $R$ (the longest distance projectile travels) and determine the launch angle of the largest range.

**1.2** (40 points) Consider the second order constant coefficient differential equation

$$2\,x'' + 5\,x' - 3\,x(t) = 0, \qquad x(0) = 3, \quad x'(0) = -2.$$

**(a)** (10 points)
Convert this equation into a system of two equations of first order and rewrite it in matrix form:

$$\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x},$$

**(b)** (10 points) Upon introducing an auxiliary dependent variable $y_3(t) = x''(t)$, convert this equation into a system of three equations of first order and rewrite it in matrix form:

$$\dot{\mathbf{y}} = \mathbf{B}\,\mathbf{y},$$

clearly identifying the 3-by-3 constant matrix $\mathbf{B}$ and 3-column vector $\mathbf{y}$.

**(c)** (10 points) Using a numerical solver, plot on the interval [0,3] solution curves from parts (a) and (b). Do you detect any discrepancy ?

**(d)** (10 points) On interval [0,3], plot the difference of functions $2\,y_3(t) + 5\,x'(t) - 3\,x(t)$ in order to control computation of the differential equation.

**1.3** (30 points) Consider a third order constant coefficient Genesio differential equation

$$x''' + a\,x'' + b\,x' + c\,x - x^2 + x^4 = 0,$$

where $a$, $b$, and $c$ are some real constants.

**(a)** (10 points)
Convert this equation into a system of first order equations and rewrite it in vector form:

$$\dot{\mathbf{y}} = \mathbf{f}\left(\mathbf{y}\right),$$

clearly identifying the 3-by-1 vector $\mathbf{f}$ and 3-column vector $\mathbf{y}$.

**(b)** (10 points) Using an appropriate software package, solve numerically the Genesio equation for $a = c = 0$ subject to some initial conditions (not identically zero).

**(c)** (10 points) Repeat the previous problem for some small values of parameters $a$ and $c$. Then plot its solution on the interval $[0, 3]$ along with the solution for $a = c = 0$.