AM 034 — Applied Mathematics - II

Brown University **Spring 2019**
Homework, Set 5 **Due April 17**

Turn this homework in by 4:45pm to the APMA 0340 homework drop-box in the lobby of the Division of Applied Mathematics, 182 George St. You can also hand in your work in your classroom at 12 (noon).

Attach this cover page to the front of your homework and staple all papers together before handing in. (Points will be deducted for failing to do so.)

Show all of your work. Correct answers without work will receive no credit.

> You are encouraged to use either free (*Maxima*, *Sage*, *SymPy*, *R*, Python, and *Octave*) or commercial (*Maple*, *Mathematica*, and MATLAB together with `MuPAD` or Live Editor) software packages.
>
> However, there are restrictions on how you may use software. You may *only* use your software to do the following things: evaluate integrals and simplify complicated algebraic expressions.

Name:

Banner ID:

(Leave this blank)

Problem	Possible	Earned
1	50	
2	5	
3	45	
Total	100	

5.1 (50 points) Consider piecewise continuous function on the interval $[-2, 2]$:

$$f(x) = \begin{cases} 1, & \text{if } -2 < x < -1, \\ x^2 - 1, & \text{if } -1 < x < 2, \end{cases}$$

(a) (10 points) Expand $f(x)$ into Fourier series.

(b) (10 points) On the interval $[-5, 5]$, plot partial Fourier sums

$$F_N(x) = \frac{a_0}{2} + \sum_{k=1}^{N} \left[a_k \cos \frac{k\pi x}{\ell} + b_k \sin \frac{k\pi x}{\ell} \right]$$

for $N = 5$, $N = 20$, and $N = 100$.

(c) (10 points) Find Cesàro approximation for $f(x)$.

(d) (10 points) On the interval $[-5, 5]$, plot partial Cesàro sums with $N = 5$, $N = 20$, and $N = 100$ terms, respectively.

(e) (10 points) Find the values of undershoot and overshoot at the points $x = -1$ and $x = 2$ of the Fourier series.

5.2 (5 points) Expand the piecewise continuous function from the previous exercise into complex Fourier series

$$f(x) \sim \sum_{n=-\infty}^{\infty} \alpha_n e^{\mathbf{j}n\pi x/\ell}.$$

5.3 (45 points) Let $f(x) = x^3 - 3x$ be defined on the interval $[0, 2]$.

(a) (10 points) Expand $f(x)$ into Fourier series on the interval $[0, 2]$.

(b) (10 points) Plot partial Fourier sums for $N = 5$, $N = 20$, and $N = 100$, on the interval $[-4, 4]$.

(c) (10 points) By expanding $f(x)$ into the interval $[-2, 0]$, find sine-Fourier series and cosine-Fourier of these expanded functions.

(d) (10 points) Plot partial sine-Fourier and cosine-Fourier sums with $N = 5$, $N = 20$, and $N = 100$ terms, respectively, on the interval $[-4, 4]$.

(e) (5 points) Which of these three Fourier series converges fastest?