AM 034 — Applied Mathematics - II

Brown University **Spring 2019**

Homework, Set 4 **Due March 20**

Turn this homework in by 4:45pm to the APMA 0340 homework drop-box in the lobby of the Division of Applied Mathematics, 182 George St. You can also hand in your work in your classroom at 12 (noon).

Attach this cover page to the front of your homework and staple all papers together before handing in. (Points will be deducted for failing to do so.)

Show all of your work. Correct answers without work will receive no credit.

> You are encouraged to use either free (*Maxima*, *Sage*, *SymPy*, *R*, Python, and *Octave*) or commercial (*Maple*™, *Mathematica*®, and MATLAB® together with `MuPAD` or Live Editor) software packages.
>
> However, there are restrictions on how you may use software. You may *only* use your software to do the following things: add matrices, multiply matrices, invert matrices, differentiate matrix-functions, find determinants, find eigenvalues, find eigenvectors, and simplify complicated algebraic expressions.

Name:

Banner ID:

(Leave this blank)

Problem	Possible	Earned
1	35	
2	35	
3	30	
Total	100	

For each of the following matrices $\mathbf{A}$,

$$\begin{bmatrix} -2 & 1 \\ 6 & 3 \end{bmatrix}, \quad \begin{bmatrix} -3 & -4 \\ 5 & 1 \end{bmatrix}, \quad \begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix}, \quad \begin{bmatrix} -5 & -2 \\ 8 & 3 \end{bmatrix}, \quad \begin{bmatrix} -6 & -8 \\ 5 & 6 \end{bmatrix}, \quad \begin{bmatrix} -2 & -4 \\ 4 & 6 \end{bmatrix}, \quad \begin{bmatrix} 7 & 1 \\ 6 & 6 \end{bmatrix}.$$

consider the homogeneous planar system $\dot{\mathbf{y}}(t) = \mathbf{A}\,\mathbf{y}(t)$.
For **each** of the seven given matrices:

4.1 (35 points) Determine the **type** of critical point at the origin

4.2 (35 points) Determine the **stability** of the critical point at the origin.

4.3 (30 points) **Plot** a phase portrait (direction field along with some solutions) to confirm your answers in parts (a) and (b).

You may find the following table useful:

Eigenvalues	**Type of Critical Point**	**Stability**
$\lambda_1 > \lambda_2 > 0$	Nodal source (node)	Unstable
$\lambda_1 < \lambda_2 < 0$	Nodal sink (node)	Asymptotically stable
$\lambda_1 < 0 < \lambda_2$	Saddle point	Unstable
$\lambda_1 = \lambda_2 > 0$, independent eigenvectors	Proper node/star point	Unstable
$\lambda_1 = \lambda_2 < 0$, independent eigenvectors	Proper node/star point	Asymptotically stable
$\lambda_1 = \lambda_2 > 0$, missing eigenvector	Improper/degenerate node	Unstable
$\lambda_1 = \lambda_2 < 0$, missing eigenvector	Improper/degenerate node	Asymptotically stable
$\lambda = \alpha \pm \mathbf{j}\beta$, $\alpha > 0$	Spiral point	Unstable
$\lambda = \alpha \pm \mathbf{j}\beta$, $\alpha < 0$	Spiral point	Asymptotically stable
$\lambda = \pm\beta\mathbf{j}$	Center	Stable