AM 034 — Applied Mathematics - II

Brown University **Spring 2019**

Homework, Set 3 **Due February 27**

Turn this homework in by 4:45pm to the APMA 0340 homework drop-box in the lobby of the Division of Applied Mathematics, 182 George St. You can also hand in your work in your classroom at 12 (noon).

Attach this cover page to the front of your homework and staple all papers together before handing in. (Points will be deducted for failing to do so.)

Show all of your work. Correct answers without work will receive no credit.

You are encouraged to use either free (*Maxima*, *Sage*, *SymPy*, *R*, Python, and *Octave*) or commercial (*Maple*, *Mathematica*, and MATLAB together with `MuPAD` or Live Editor) software packages.

However, there are restrictions on how you may use software. You may *only* use your software to do the following things: add matrices, multiply matrices, invert matrices, differentiate matrix-functions, find determinants, find eigenvalues, find eigenvectors, and simplify complicated algebraic expressions.

Name:

Banner ID:

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Problem	Possible	Earned
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

3.1 (20 points) Which of the following matrices is diagonalizable ?

$$(\mathbf{a}) \quad \begin{bmatrix} -731 & 228 & 690 \\ -410 & 131 & 385 \\ -650 & 202 & 614 \end{bmatrix}, \qquad (\mathbf{b}) \quad \begin{bmatrix} -41 & 18 & -6 \\ -126 & 55 & -18 \\ -63 & 27 & -8 \end{bmatrix}, \qquad (\mathbf{c}) \quad \begin{bmatrix} -14 & 6 & 0 \\ -24 & 10 & 3 \\ 33 & -15 & 10 \end{bmatrix}.$$

Find minimal polynomials for each of the above matrices.

3.2 (20 points) For matrices $\mathbf{A}$ and $\mathbf{B}$ from parts (a) and (b) of the previous exercise, find

$$\mathbf{\Phi}_A(t) = \frac{\sin\left(\sqrt{\mathbf{A}}\, t\right)}{\sqrt{\mathbf{A}}}, \quad \mathbf{\Phi}_B(t) = \frac{\sin\left(\sqrt{\mathbf{B}}\, t\right)}{\sqrt{\mathbf{B}}}, \quad \mathbf{\Psi}_A(t) = \cos\left(\sqrt{\mathbf{A}}\, t\right), \quad \mathbf{\Psi}_B(t) = \cos\left(\sqrt{\mathbf{B}}\, t\right).$$

3.3 (20 points) For each matrix from the previous exercise, show that the matrix-functions $\mathbf{\Phi}(t)$ and $\mathbf{\Psi}(t)$ satisfy the matrix differential equation

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\, \mathbf{\Phi}_A(t) + \mathbf{A}\mathbf{\Phi}(t) = \mathbf{0} \qquad \text{or} \qquad \frac{\mathrm{d}^2}{\mathrm{d}t^2}\, \mathbf{\Psi}_A(t) + \mathbf{A}\, \mathbf{\Psi}(t) = \mathbf{0}.$$

(Similar equations are valid for matrix $\mathbf{B}$.) For each matrix $\mathbf{A}$ and $\mathbf{B}$, what initial conditions do these matrix-functions $\mathbf{\Phi}(t)$ and $\mathbf{\Psi}(t)$ satisfy?

3.4 (20 points) For the matrices $\mathbf{A} = \begin{bmatrix} -731 & 228 & 690 \\ -410 & 131 & 385 \\ -650 & 202 & 614 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} -41 & 18 & -6 \\ -126 & 55 & -18 \\ -63 & 27 & -8 \end{bmatrix}$, construct the exponential matrices

$$\mathbf{U}_A(t) = e^{\mathbf{A}\, t} \qquad \text{and} \qquad \mathbf{U}_B(t) = e^{\mathbf{B}\, t}.$$

Show that these matrix-functions are (unique) solutions of the following matrix initial value problems:

$$\frac{\mathrm{d}}{\mathrm{d}t}\, \mathbf{U}_A(t) = \mathbf{A}\, \mathbf{U}_A(t), \quad \mathbf{U}_A(0) = \mathbf{I} \qquad \text{and} \qquad \frac{\mathrm{d}}{\mathrm{d}t}\, \mathbf{U}_B(t) = \mathbf{B}\, \mathbf{U}_B(t), \quad \mathbf{U}_B(0) = \mathbf{I},$$

respectively.

3.5 (20 points) Find square roots of the matrix $\mathbf{C} = \begin{bmatrix} -14 & 6 & 0 \\ -24 & 10 & 3 \\ 33 & -15 & 10 \end{bmatrix}$.