AM 034 — Applied Mathematics - II

Brown University **Spring 2019**

Homework, Set 2 **Due February 20**

Turn this homework either in class or in to the APMA 0340 homework drop-box in the lobby of the Division of Applied Mathematics, 182 George St.

Attach this cover page to the front of your homework and staple all papers together before handing in. (Points will be deducted for failing to do so.)

Show all of your work. Correct answers without work will receive no credit.

Name:

Banner ID:

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Problem	**Possible**	**Earned**
1	20	
2	20	
3	40	
4	20	
Total	100	

2.1 (20 points) For given two 3×3 matrices $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$ and $\mathbf{S} = \begin{bmatrix} 4 & 2 & 5 \\ 5 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, calculate the matrix $\mathbf{B} = \mathbf{S}^{-1}\mathbf{A}\mathbf{S}$. Then determine the eigenvalues and corresponding eivenvectors for matrices $\mathbf{A}$ and $\mathbf{B}$.

2.2 (20 points) For what vectors $\mathbf{b}$ does the linear system of equations $\mathbf{A}\mathbf{x} = \mathbf{b}$ has a nontrivial solution?

$$\mathbf{A} = \begin{bmatrix} 2 & 5 & 4 \\ 1 & 3 & 2 \\ 2 & 1 & 4 \end{bmatrix}.$$

2.3 ($7 \times 4 + 12 = 40$ points) For each of the following matrices,

$$(\mathbf{a}) \quad \begin{bmatrix} -37 & -20 & -70 \\ 106 & 56 & 185 \\ -12 & -6 & -17 \end{bmatrix}, \qquad (\mathbf{b}) \quad \begin{bmatrix} 2 & 3 & 8 \\ 1 & 2 & -1 \\ 1 & 3 & -1 \end{bmatrix}, \qquad (\mathbf{c}) \quad \begin{bmatrix} 17 & 8 & 24 \\ -42 & -20 & -63 \\ 4 & 2 & 7 \end{bmatrix},$$

find $\mathbf{\Phi}(t) = e^{\mathbf{A}t}$ by performing the following steps:

(a) determine all eigenvalues and corresponding eigenvectors;

(b) Form a 3×3 matrix $\mathbf{S}$, where each column is an eigenvector;

(c) Calculate the diagonal matrix $e^{\mathbf{D}t}$ having on diagonal $e^{\lambda_j t}$, where λ_j $(j = 1, 2, 3)$ is an eigenvalue;

(d) Find $\mathbf{\Phi}(t) = e^{\mathbf{A}t} = \mathbf{S}\, e^{\mathbf{D}t}\, \mathbf{S}^{-1}$.

(e) ($4 \times 3 = 12$ points) Show that $\mathbf{\Phi}(t)$ for each matrix is a solution of matrix differential equation subject to the initial condition:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{\Phi}(t) = \mathbf{A}\,\mathbf{\Phi}(t), \qquad \mathbf{\Phi}(0) = \mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

2.4 ($10 + 10 = 20$ points) Given the matrices $\mathbf{A}$ and $\mathbf{B}$ as shown below. Find eigenvalues of their products $\mathbf{A}\,\mathbf{B}$ and $\mathbf{B}\,\mathbf{A}$ and determine their corresponding eigenvectors. Is matrix $\mathbf{B}\,\mathbf{A}$ diagonalizable?

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 3 & 2 \\ 2 & 5 & 1 & -1 \end{bmatrix} \qquad \text{and} \qquad \mathbf{B} = \begin{bmatrix} 2 & 1 \\ -1 & 3 \\ -3 & 1 \\ 2 & 3 \end{bmatrix}.$$