AM 034 — Applied Mathematics - II

Brown University **Spring 2019**

Homework, Set 1 **Due February 13**

Turn this homework in by 4:45pm to the APMA 0340 homework drop-box in the lobby of the Division of Applied Mathematics, 182 George St. You can also hand in your work in your classroom at 12 (noon).

Attach this cover page to the front of your homework and staple all papers together before handing in. (Points will be deducted for failing to do so.)

Show all of your work. Correct answers without work will receive no credit.

> You are encouraged to use either free (*Maxima*, *Sage*, *SymPy*, *R*, Python, and *Octave*) or commercial (*Maple*, *Mathematica*, and MATLAB together with `MuPAD` or Live Editor) software packages.
>
> However, there are restrictions on how you may use software. You may *only* use your software to do the following things: evaluate integrals and simplify complicated algebraic expressions.

Name:

Banner ID:

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Problem	**Possible**	**Earned**
1	30	
2	40	
3	30	
Total	100	

1.1 (30 points) Find a first order system that is equivalent to the second order equation named after the German scientist Georg Duffing (1861–1944)

$$\ddot{y} + 2\eta\,\dot{y} + y + \varepsilon\,y^3 = \cos(\omega t),$$

where the dots denote differentiation of $y(t)$ with respect t: $\dot{y} = \mathrm{d}y/\mathrm{d}t$. Here $\eta \geq 0$ and ε are physical parameters, and ω is given frequency of excitation. In general, the Duffing equation does not admit an exact symbolic solution.

1.2 (40 points) The initial value problem for the vector differential equation

$$\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} = \mathbf{f}(t, \mathbf{u}), \qquad \mathbf{u}(0) = \mathbf{u}_0,$$

where $\mathbf{u}$ is unknown column-vector and $\mathbf{f}$ is a given column vector, is equivalent to the integral equation

$$\mathbf{u}(t) = \mathbf{u}_0 + \int_0^t \mathbf{f}(s, \mathbf{u}(s))\,\mathrm{d}s.$$

Its solution can be found as the limit $\mathbf{u}(t) = \lim\limits_{n\to\infty} \boldsymbol{\phi}_n(t)$, where the sequence of vector functions is defined recursively by

$$\boldsymbol{\phi}_0 = \mathbf{u}_0, \qquad \boldsymbol{\phi}_{n+1}(t) = \mathbf{u}_0 + \int_0^t \mathbf{f}\left(s, \boldsymbol{\phi}_n(s)\right)\mathrm{d}s, \qquad n = 0, 1, 2, \ldots.$$

Consider the initial value problem for the homogeneous Duffing equation:

$$\ddot{y} + 2\eta\,\dot{y} + y + \varepsilon\,y^3 = 0, \qquad y(0) = d, \quad \dot{y}(0) = v,$$

where $d = 1$ is the initial displacement and $v = -1$ is the initial velocity.

Convert the above initial value problem to the system of first order equations for the following numerical values: $\eta = 3$ and $\varepsilon = 1$. Then using Picard's iteration, find the first four iterative terms $\boldsymbol{\phi}_1(t), \boldsymbol{\phi}_2(t), \boldsymbol{\phi}_3(t), \boldsymbol{\phi}_4(t)$ for the Duffing system of equations. Finally, determine the corresponding four approximations to the solution $y(t)$ of the original second order Duffing equation.

1.3 (30 points) Consider a third order constant coefficient differential equation

$$y''' - 3\,y'' + 7\,y' - 2\,y = e^{-|t|}\cos 2t.$$

Convert this equation into a system of first order equations and rewrite it in matrix form:

$$\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{f}(t),$$

clearly identifying the 3-by-3 constant matrix $\mathbf{A}$ and 3-column vector $\mathbf{f}$.