AM 034 — Applied Mathematics - II

Brown University **Fall 2021**

Homework, Set 2 **Due September 29**

2.1 (30 points) Consider the initial value problem

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}, \qquad \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 7 \\ -7 \end{bmatrix}.$$

(a) (10 points) Write down an integral equation that is equivalent to the given initial value problem.

(b) (20 points) Using Picard's approximation, find four-term approximation to the given initial value problem.

2.2 (50 points) Consider a simple pendulum equation

$$\ddot{\theta} + \omega^2 \sin\theta = 0, \qquad \theta(0) = \frac{\pi}{3}, \quad \dot{\theta}(0) = -\frac{1}{10}.$$

(a) (10 points) Convert the given initial value problem to a system of first order differential equations suitable for Picard's iteration (so the right-hand side is a polynomial).

(b) (10 points) Write down an integral equation that is equivalent to the given initial value problem.

(c) (10 points) Set up a Picard iteration scheme based on part (b).

(d) (10 points) Perform three Picard's iterations to find a polynomial approximation to $\theta(t)$.

(e) (10 points) On interval $[0, 0.6]$, plot your Picard's approximation along with the solution curve obtained upon application of a solution solver to the given initial value problem.

2.3 (20 points) Suppose you are given a matrix with variable entries:

$$\mathbf{A}(t) = \begin{bmatrix} t & t^2 \\ 1/t & 2-t \end{bmatrix}.$$

Find the derivative of its square $\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{A}^2(t)$. Does a regular power rule hold ?