AM 034 — Applied Mathematics - II

Brown University **Fall 2021**
Homework, Set 1 **Due September 22**

1.1 (30 points) Suppose that a projectile is launched into the air from ground with initial velocity v_0 and angle α with the horizontal plane. Assuming that there is no air resistance, the model for its movement is as follows

$$\begin{aligned} &\ddot{x} = 0, && x(0) = 0, \quad \dot{x}(0) = v_0 \cos\alpha, \\ &\ddot{y} = -g; && y(0) = 0, \quad \dot{y}(0) = v_0 \sin\alpha. \end{aligned}$$

Here $g \approx 9.81$ is the acceleration due to gravity and overdot denotes the derivative with respect to time t.

(a) (5 points) Show that the projectile's trajectory is a parabola.

(b) (5 points) Confirm part (a) by plotting several trajectories on an $x - y$ plane.

(c) (10 points) Let (x_m, y_m) be coordinates of the vertex corresponding to the parabola solution curve, so y_m is the maximum height attained and x_m is the corresponding horizontal coordinate. Plot the curve with coordinates (x_m, y_m) for different launch angles α.

(d) (10 points) Find the formula for the range R (the longest distance projectile travels) and determine the launch angle of the largest range.

1.2 (40 points) Consider the second order constant coefficient differential equation

$$2\,x'' + 5\,x' - 3\,x(t) = 0, \qquad x(0) = 3, \quad x'(0) = -2.$$

(a) (10 points)
Convert this equation into a system of two equations of first order and rewrite it in matrix form:

$$\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x},$$

(b) (10 points) Upon introducing an auxiliary dependent variable $y_3(t) = x''(t)$, convert this equation into a system of three equations of first order and rewrite it in matrix form:

$$\dot{\mathbf{y}} = \mathbf{B}\,\mathbf{y},$$

clearly identifying the 3-by-3 constant matrix $\mathbf{B}$ and 3-column vector $\mathbf{y}$.

(c) (10 points) Using a numerical solver, plot on the interval [0,3] solution curves from parts (a) and (b). Do you detect any discrepancy ?

(d) (10 points) On interval [0,3], plot the difference of functions $2\,y_3(t) + 5\,x'(t) - 3\,x(t)$ in order to control computation of the differential equation.

1.3 (30 points) Consider a third order constant coefficient Genesio differential equation

$$x''' + a\,x'' + b\,x' + c\,x - x^2 + x^4 = 0,$$

where a, b, and c are some real constants.

(a) (10 points)
Convert this equation into a system of first order equations and rewrite it in vector form:

$$\dot{\mathbf{y}} = \mathbf{f}\,(\mathbf{y})\,,$$

clearly identifying the 3-by-1 vector $\mathbf{f}$ and 3-column vector $\mathbf{y}$.

(b) (10 points) Using an appropriate software package, solve numerically the Genesio equation for $a = c = 0$ subject to some initial conditions (not identically zero).

(c) (10 points) Repeat the previous problem for some small values of parameters a and c. Then plot its solution on the interval $[0, 3]$ along with the solution for $a = c = 0$.