Week 10

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Method of Undetermined Coefficients

The method of undetermined coefficients is used to find a particular solution for a linear differential equation with constant coefficients

$$L[\mathtt{D}]\, y(x) = f(x),$$

where $L[\mathtt{D}]$ is a linear differential operator

$$L[\mathtt{D}] = a_m \mathtt{D}^m + a_{m-1}\mathtt{D}^{m-1} + \cdots + a_1 \mathtt{D} + a_0 \mathtt{I}, \qquad \mathtt{D} = \frac{\mathrm{d}}{\mathrm{d}x},$$

and $f(x)$ is a function that is annihilated by some constant coefficient differential operator

$$M[\mathtt{D}]\, f(x) = 0.$$

Therefore, $f(x)$ is polynomial times exponential function times a trigonometric function. The roots of the characteristic polynomial

$$M[\sigma] = 0$$

are called the **control numbers**. If we multiply the given nonhomogeneous differential equation $L[\mathtt{D}]\, y(x) = f(x)$ by the differential operator $M[\mathtt{D}]$, we reduce the problem for nonhomogeneous equation to a similar problem for a homogeneous equation:

$$M[\mathtt{D}]\, L[\mathtt{D}]\, y = 0.$$

If a control number σ does not match the roots of the characteristic polynomial $L(\lambda) = 0$, then a particular solution has the same form as $f(x)$. If the control number σ matches the root of multiplicity m of the characteristic equation, then a particular solution of the nonhomogeneous equation has the same form as $f(x)$ multiplied by x^m. The particular values of coefficients of a particular solution are determined upon substitution into the given nonhomogeneous equation.

Examples of application of the method are given on the web site:
http://www.cfm.brown.edu/people/dobrush/am33/Mathematica/ch4/muc.html