

Numerical Methods Project

In general, we are using numerical methods to approximate the solution to

$$u_t = f(t, u(t)).$$

1. Use
 - (a) Euler's method
 - (b) Second order Runge-Kutta
 - (c) Fourth order Runge Kutta
 - (d) any multi-step method you choose

to approximate the solution to the differential equation

$$u_t = \cos(\pi t) + u(t)$$

with initial condition

$$u(0) = 2.$$

Solve this for time t up to 2.0.

Show a graph of the solution vs. the exact solution, and a graph of the errors for number of points $N = 10, 20, 40, 80, 160, 320, 640$.

Show the effect of increasing the number of points on the error (using the different norms. What can you conclude about the order of the method? Comment on the behavior you see and determine which method performed best.

2. Use
 - (a) Euler's method
 - (b) Implicit Euler's method
 - (c) Second order Runge-Kutta
 - (d) Fourth order Runge Kutta
 - (e) any explicit multi-step method you choose
 - (f) any implicit method you choose

to approximate the solution to the differential equation

$$u_t = -400 * u(t)$$

with initial condition

$$u(0) = .2$$

Solve this for time t up to 1.0, or as long as you still get reasonable answers.

Show a graph of the solution vs. the exact solution, and a graph of the errors for number of points $N = 10, 20, 40, 80, 160, 320, 640$. Show the effect of increasing the number of points on the error (using the different norms). What can you conclude about the order of the method? Comment on the behavior you see and determine which method performed best. Explain the differences between the implicit and explicit methods.

3. Use

- (a) Euler's method
- (b) Second order Runge-Kutta
- (c) Fourth order Runge Kutta

to approximate the solution to the differential equation

$$u_t = \begin{pmatrix} 0 & 1 \\ -6 & 5 \end{pmatrix} u$$

with initial condition

$$u(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Solve this for time t up to 2.0.

Show a graph of the solution vs. the exact solution, and a graph of the errors for number of points $N = 10, 20, 40, 80, 160, 320, 640$.

Show the effect of increasing the number of points on the error (using the different norms. What can you conclude about the order of the method? Comment on the behavior you see and determine which method performed best.

For each problem, a good presentation of data would include:

1. A description of the problem and numerical method
2. One table showing the errors resulting from all the different methods
3. One graph showing the numerical solutions
4. One graph showing the errors
5. A writeup explaining your results and analysis of what is happening with the different methods and why
6. A copy of your codes in the appendix