the error in this formula has the form $\sum_{n=1}^{\infty} a_n h^{2n}$. Determine the coefficients $a_{2n}$ explicitly. Also derive the error term given in Equation (9).

6. Derive the following two formulas for approximating derivatives and show that they are both $\mathcal{O}(h^3)$ by establishing their error terms:

$$ f'(x) = \frac{1}{12h} \left[ -f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h) \right] $$

$$ f''(x) = \frac{1}{12h^2} \left[ -f(x+2h) + 16f(x+h) - 30f(x) + 16f(x-h) - f(x-2h) \right] $$

7. Derive the following two formulas for approximating the third derivative. Find their error terms. Which formula is more accurate?

$$ f'''(x) = \frac{1}{h^3} \left[ f(x + 3h) - 3f(x + 2h) + 3f(x + h) - f(x) \right] $$

$$ f'''(x) = \frac{1}{2h^3} \left[ f(x + 2h) - 2f(x + h) + 2f(x - h) - f(x - 2h) \right] $$

8. Carry out the instructions in Problem 7 for the following fourth-derivative formulas:

$$ f^{(4)} = \frac{1}{h^4} \left[ f(x + 4h) - 4f(x + 3h) + 6f(x + 2h) - 4f(x + h) + f(x) \right] $$

$$ f^{(4)} = \frac{1}{h^4} \left[ f(x + 2h) - 4f(x + h) + 6f(x) - 4f(x - h) + f(x - 2h) \right] $$

9. Show that in Richardson extrapolation,

$$ D(2, 2) = \frac{16}{15} \psi(h/2) - \frac{1}{15} \psi(h) $$

10. Show how to use Richardson extrapolation employing $x_n$ and $x_{n/2}$ if

$$ L = x_n + a_1 n^{-1} + a_2 n^{-2} + a_3 n^{-3} + \cdots $$

11. Prove or disprove:

(a) If $L - x_n = \mathcal{O}(n^{-1})$, then $L - (2x_2 - x_n) = \mathcal{O}(n^{-2})$.

(b) If $L - x_n = \mathcal{O}(n^{-1})$, then $L - x_{n/2} = \mathcal{O}(n^{-2})$.

Discuss the numerical consequences of this problem.

12. Show how to use Richardson extrapolation if

$$ L = \psi(h) + a_1 h + a_2 h^3 + a_3 h^5 + \cdots $$

13. Suppose that $L = \lim_{h \to 0} f(h)$ and that $L - f(h) = c_0 h^6 + c_0 h^9 + \cdots$. We assume that $L$ is a combination of $f(h)$ and $f(h/2)$ should be the best estimate of $L$. 