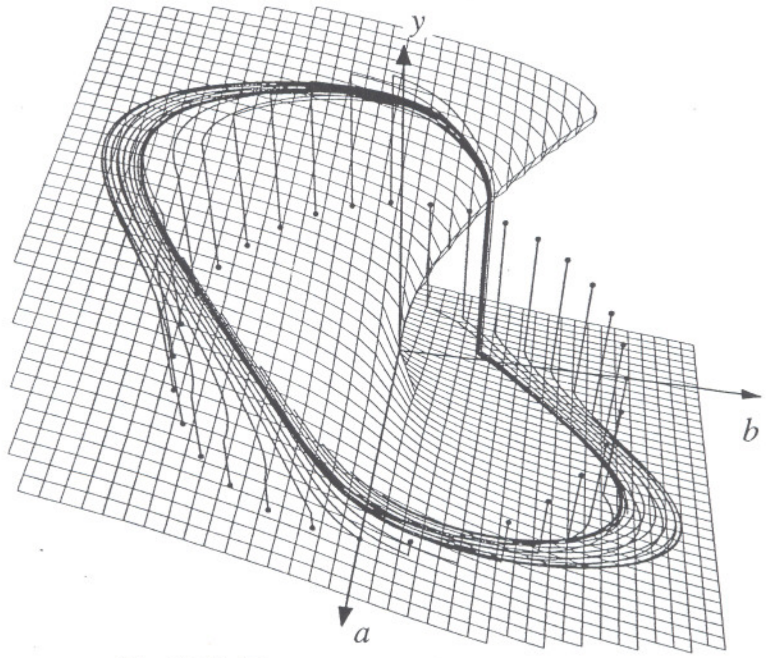

Fig. 10.3. The cusp catastrophe with $N = 32$.

CUSP — this is a combination of Zeeman's "cusp catastrophe" model ($-\varepsilon\dot{y} = y^3 + ay + b$) for the nerve impulse mechanism (Zeeman 1972) combined with the van der Pol oscillator (see Fig. 10.3)

$$\begin{aligned}\frac{\partial y}{\partial t} &= -\frac{1}{\varepsilon}(y^3 + ay + b) + \sigma\frac{\partial^2 y}{\partial x^2}\\ \frac{\partial a}{\partial t} &= b + 0.07v + \sigma\frac{\partial^2 a}{\partial x^2}\\ \frac{\partial b}{\partial t} &= (1-a^2)b - a - 0.4y + 0.035v + \sigma\frac{\partial^2 b}{\partial x^2}\end{aligned} \tag{10.8}$$

where

$$v = \frac{u}{u+0.1}, \quad u = (y-0.7)(y-1.3).$$

We put $\sigma = 1/144$ and make the problem stiff by choosing $\varepsilon = 10^{-4}$. We discretize the diffusion terms by the method of lines

$$\begin{aligned}\dot{y}_i &= -10^4(y_i^3 + a_i y_i + b_i) + D(y_{i-1} - 2y_i + y_{i+1})\\ \dot{a}_i &= b_i + 0.07v_i + D(a_{i-1} - 2a_i + a_{i+1}) \qquad i = 1,\ldots,N\\ \dot{b}_i &= (1-a_i^2)b_i - a_i - 0.4y_i + 0.035v_i + D(b_{i-1} - 2b_i + b_{i+1})\end{aligned} \tag{10.8'}$$

where

$$N = 32, \quad v_i = \frac{u_i}{u_i + 0.1}, \quad u_i = (y_i - 0.7)(y_i - 1.3), \quad D = \sigma N^2 = \frac{N^2}{144},$$

with periodic boundary conditions

$$\begin{aligned}y_0 &:= y_N, & a_0 &:= a_N, & b_0 &:= b_N,\\ y_{N+1} &:= y_1, & a_{N+1} &:= a_1, & b_{N+1} &:= b_1,\end{aligned}$$