# Fast Parallel Direct Solvers For Coarse Grid Problems

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August 11, 1997

**Abstract**

We develop a fast direct solver for parallel solution of "coarse grid" problems, $A\underline{x} = \underline{b}$, such as arise when domain decomposition or multigrid methods are applied to elliptic partial differential equations in $d$ space dimensions. The approach is based upon a (quasi-) sparse factorization of the *inverse* of $A$. If $A$ is $n \times n$ and the number of processors is $P$, our approach requires $O(n^{\gamma} \log_2 P)$ time for communication and $O(n^{1+\gamma}/P)$ time for computation, where $\gamma \equiv \frac{d-1}{d}$. Results from a 512 node Intel Paragon show that our algorithm compares favorably to more commonly used approaches which require $O(n \log_2 P)$ time for communication and $O(n^{1+\gamma})$ or $O(n^2/P)$ time for computation. Moreover, for leading edge multicomputer systems with thousands of processors and $n = P$ (i.e., communication dominated solves), we expect our algorithm to be markedly superior as it achieves substantially reduced message volume and arithmetic complexity over competing methods while retaining minimal message startup cost.

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