Fast Parallel Direct Solvers For Coarse Grid Problems

H. M. Tufo
P. F. Fischer *

May 23, 1997

Abstract

We develop a fast direct solver for parallel solution of "coarse grid" problems, $A\underline{x} = \underline{b}$, such as arise when domain decomposition or multigrid methods are applied to elliptic partial differential equations in d space dimensions. The approach is based upon a (quasi-) sparse factorization of the *inverse* of A. If the dimension of the system is n and the number of processors is P, our approach requires $O(n^{\gamma} \log_2 P)$ time for communication and $O(n^{1+\gamma}/P)$ time for computation, where $\gamma \equiv \frac{d-1}{d}$. Results from a 512 node Intel Paragon show that our algorithm compares favorably to more commonly used approaches which require $O(n \log_2 P)$ time for communication and $O(n^{1+\gamma})$ or $O(n^2/P)$ time for computation. Moreover, for leading edge multicomputer systems with thousands of processors and $n = P$ (i.e., communication dominated solves), we expect our algorithm to be markedly superior as it achieves substantially reduced message volume and arithmetic complexity over current competing methods while retaining minimum message startup cost.

Presented at the 13th annual conference of the *Intel Supercomputer Users Group*, Sandia National Laboratory, Albuquerque, New Mexico, June 13, 1997.

*Division of Applied Mathematics, Brown University, Providence, RI 02912, USA. E-mail: *pff@cfm.brown.edu, hmt@cfm.brown.edu*