Applied Math 9
Problem Set 2 for Markov Chains

1. Consider once again the Markov chain with one step transition probabilities
$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & .5 & .5 \\ 0 & 0 & .5 & .5 \end{pmatrix}.$$
Compute the distribution at time $n = 100$, given that the chain starts at state 3 at time zero. What is the distribution if you start at state 2?

2. Now consider a slight modification of the one step probabilities:
$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ .99 & 0 & .01 & 0 \\ 0 & 0 & .5 & .5 \\ .02 & 0 & .49 & .49 \end{pmatrix}.$$
Repeat the calculations of problem 1. Are the answers close? Why or why not?

3. Suppose the process with one step transitions
$$\begin{pmatrix} 0 & .4 & 0 & .6 & 0 & 0 \\ 0 & .5 & .5 & 0 & 0 & 0 \\ 0 & .5 & .5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ .05 & 0 & 0 & 0 & .45 & .5 \\ 0 & 0 & 0 & 0 & .1 & .9 \end{pmatrix}$$
starts at state 6. Without making any calculations, can you tell what happens to the process as time goes to infinity? Do you think this process has one or more than one invariant distribution?

4. One bit of data is to travel through a N nodes of a communication network. The data is binary, i.e., it takes only the values 0 and 1. At each node there is a probability p that the data is corrupted, i.e., a 0 is turned into a 1 or a 1 is turned into a 0. Can you identify a Markov process that describes the travel of the data bit through the network? What (implicit) assumptions are you making for the process to be Markov?