Applied Math 9
Problem Set 1 for Math of Music

1. *Partial derivatives.* Recall that in class we discussed the notion of a partial derivative. In particular, for a smooth function $u(x,t)$ defined on some open rectangle
$$(a,b) \times (c,d)$$
in two dimensional space, the partial derivative with respect to x at a point (x,t) in this rectangle is defined to be
$$u_x(x,t) = \frac{\partial}{\partial x} u(x,t) = \lim_{\delta \to 0} \frac{u(x+\delta,t) - u(x,t)}{\delta},$$
and the partial derivative with respect to t (denoted $u_t(x,t)$) is defined similarly. One can compute the partial with respect to x by pretending that t is a constant and computing the ordinary derivative with respect to x, and likewise for the partial with respect to t. Compute the partials, both with respect to x and t, for the indicated functions on the indicated rectangles.
$$\begin{array}{ll} \sin xt, & (x,t) \in (-\infty,\infty) \times (-\infty,\infty) \\ \sin(x^2+t^2), & (x,t) \in (-\infty,\infty) \times (-\infty,\infty) \\ \log(x/t), & (x,t) \in (0,\infty) \times (0,\infty) \\ \cos x \cos t, & (x,t) \in (-\infty,\infty) \times (-\infty,\infty). \end{array}$$

2. Suppose that $L > 0$ is a constant. Show that for any value of λ the function
$$f(x) = \sin \lambda x$$
is a solution to the *ordinary differential equation*
$$f_{xx}(x) + \lambda^2 f(x) = 0.$$
Next find that particular constants $\lambda > 0$ such that f not only satisfies this differential equation, but also the boundary conditions
$$f(0) = 0, f(L) = 0.$$

3. Recall that two n-dimensional vectors $\mathbf{y}$ and $\mathbf{z}$ are said to be orthogonal (or perpendicular) if the dot product is zero:
$$\mathbf{y} \cdot \mathbf{z} = \sum_{i=1}^{n} y_i z_i = 0.$$

We use a similar definition if we want to speak of two continuous functions defined on the interval $[0, L]$ as being orthogonal. The inner product of two continuous functions f and g mapping $[0, L]$ to the real numbers is defined to be

$$\langle f, g \rangle = \frac{1}{L} \int_0^L f(x)g(x)dx.$$

Show that the following functions are all mutually orthogonal on $[0, L]$:

$$\sin \frac{n\pi x}{L}, n = 1, 2, \ldots.$$

4. Show also that this inner product possesses the familiar linearity of the ordinary dot product. In other words, if f, g and h are continuous functions mapping $[0, L]$ to the reals and if a and b are real numbers, then

$$\langle af + bg, h \rangle = a \langle f, h \rangle + b \langle g, h \rangle .$$