Applied Math 9

Problem Set 5 for Zero Sum Games

1. Recall that an $n \times n$ dimensional matrix A defines a linear mapping from n dimensional space to itself via $\mathbf{y} = A\mathbf{x}$. Which of the following mappings are linear, and hence representable by a matrix? For the linear mappings, find the corresponding matrix.

$$\begin{array}{rclcl} \mathbf{y}^T &=& T(x_1, x_2, x_3) &=& (x_1 + x_2 + x_3, 0, 0) \\ \mathbf{y}^T &=& T(x_1, x_2, x_3) &=& (x_2, x_3, x_1) \\ \mathbf{y}^T &=& T(x_1, x_2, x_3) &=& (x_1 + x_2 + x_3, 1, -1) \\ \mathbf{y}^T &=& T(x_1, x_2, x_3) &=& (x_1 x_2 x_3, 0, 0) \end{array}.$$

2. In class we noted a fundamental result, which is that when an $n \times n$ dimensional matrix A has nonzero determinant, then the equation $\mathbf{y} = A\mathbf{x}$ always has a solution. This means that the mapping from $\mathbf{x}$ to $\mathbf{y}$ is one to one and onto, and so the mapping has an *inverse.* We will use Matlab to calculate inverses, and later on mention the technique of *Gaussian elimination* to compute an inverse by hand. However, it would be nice to know if the inverse mapping is linear, so that we can describe it by a matrix. What do you think?

3. What is the determinant of each of the following $n \times n$ square matrices?

$$A = (a_{ij}), \text{ where } a_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

(the so-called *identity* matrix, usually written I),

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & -1 & -3 \end{pmatrix}$$

(a particular *lower triangular* matrix),

$$A = (a_{ij}), \text{ where } a_{ij} = \begin{cases} a_i & i = j \\ 0 & i < j \end{cases}$$

(a general lower triangular matrix). You should express the determinant for the last matrix in terms of the a_i. Note that not all the entries of the matrix are specified.

4. (Games in a random environment.) Consider the following game. A referee tosses a coin, and show the outcome to Player 1. Player 1 can either (i) pass and pay \$5 to Player 2, or (ii) continue. In case (i), the game terminates. In case (ii), Player 2 can either (i) pass, and pay Player 1 \$5, or (ii) call. In case (ii), heads means Player 2 pays \$10 to Player 1, and tails means Player 1 pays \$10 to Player 2. What are the pure strategies for the two players? Here, a pure strategy for Player 1 is an action taken after seeing the coin. Thus Player 1 has four pure strategies. Player 2 has two pure strategies, which depend on the action of Player 1. Write down the payoff matrix, using expected values for entries whose outcome depends on the coin.