APPLIED MATH 9

Answer to Computational Problem Set 2 for Zero Sum Games

1. For arbitrary k, the pay-off matrix is

$$B = \begin{pmatrix} 1 & 1 & 1-k \\ 2-k & 2 & 2 \\ 3 & 3-k & 3 \end{pmatrix}$$

Matlab function *matrixk.m* is

```
function B=matrixk(k)
B=[1 1 1-k
2-k 2 2
3 3-k 3];
```

2.

$$B = \left(\begin{array}{ccc|c} 1 & 1 & .5 & .5 \\ 1.5 & 2 & 2 & 1.5 \\ 3 & 2.5 & 3 & 2.5 \\ \hline 3 & 2.5 & 3 & 2.5 \end{array}\right)$$

The upper game minmax equals the lower game maxmin which is 2.5, thus the saddle point is $b_{3,2}$. The value of the game is 2.5. Optimal strategies for player 1 are to show 3 fingers or $p_1 = [001]$ and for player 2 to show 2 fingers or $p_2 = [010]^T$.

3.

```
function [p,z]=uppergamelp(B)

% B is payoff matrix which can be any m by n.
% This sovles the uppergame of B as a LP problem.
% maximize z under p*B >= z and sum(p) = 1, p > 0
% or minimize c*[pz]' under [-B'1; -10; 10][pz]' <= b

[m,n] = size(B);

B=[-B' ones(n,1); [-ones(1,m) 0;ones(1,m) 0]];
```

```
b=[zeros(n,1);-1;1];
c=[zeros(m,1);-1]';
lb=[zeros(m,1);-inf];

p=linprog(c,B,b,[ ],[ ],lb);
z=p(end); p=p(1:end-1);
```

```
function [p,z]=lowergamelp(B)
```

$[m, n] = size(B);$

```
B=[B -ones(m,1); [-ones(1,n) 0;ones(1,n) 0]];

b=[zeros(m,1);-1;1];
c=[zeros(n,1);1]';
lb=[zeros(n,1);-inf];

p=linprog(c,B,b,[ ],[ ],lb);
z=p(end); p=p(1:end-1);
```

Input in matlab:

```
>> B=matrixk(6)

B =

1 1 -5
-4 2 2
3 -3 3

>> [p,z]=lowergamelp(B)
Optimization terminated successfully.

p =
```

0.3333
0.5000
0.1667

z =

4.2633e-13

>> [p,z]=uppergamelp(B)
Optimization terminated successfully.

p =

0.3333
0.3333
0.3333

z =

-5.1159e-13

4. Repeat the computation with matlab for $k = 1/2$,
>> B=matrixk(.5)

B =

1.0000 1.0000 0.5000
1.5000 2.0000 2.0000
3.0000 2.5000 3.0000

>> [p,z]=uppergamelp(B)
Optimization terminated successfully.

p =

0.0000
0.0000

```
1.0000

z =

2.5000

>> [p,z]=lowergamelp(B)
Optimization terminated successfully.

p =

0.0000
1.0000
0.0000

z =

2.5000
```

5. Show (using matlab or simply by a brief inspection of the matrix) that, if $k = 2$, there exist optimum strategies in which Player 1 never shows one finger and Player 2 never shows three fingers.

```
>> B=matrixk(2)

B =

1 1 -1
0 2 2
3 1 3

>> [p,z]=uppergamelp(B)
Optimization terminated successfully.

p =

0.0000
0.5000
```

0.5000

z =

1.5000

>> [p,z]=lowergamelp(B)
Optimization terminated successfully.

p =

0.2500
0.7500
0.0000

z =

1.5000

which shows that player 1 never shows one finger and player 2 never shows three figners, since the probability of showing it is zero for their optimal strategies. Or we can inspect the matrix B: Player 1 will never show one finger because showing three fingers is always at least as good from his point of view. Knowing that Player 1 will never show one finger, player 2 should never show three fingers because showing two fingers is at least as good from his point of view.