Applied Math 9
Handout 5 for Zero Sum Games: Converting the Maxmin Problem to a Max Problem

We continue with the Holmes and Moriarty example, so the payoff matrix is

$$A = \begin{pmatrix} 0 & .5 \\ 1 & 0 \end{pmatrix}.$$

Consider the lower game. This has the value

$$v_l = \max_{p_1+p_2=1, p_1\geq 0, p_2 \geq 0} \ \min_{r_1+r_2=1, r_1\geq 0, r_2\geq 0} \sum_{i=1}^{2}\sum_{j=1}^{2} a_{ij} p_i r_j.$$

A particular choice of p is optimal if and only if

$$\sum_{i=1}^{2}\sum_{j=1}^{2} a_{ij} p_i r_j \geq v_l \tag{1}$$

for all $r = (r_1, r_2)$. If we take $r = (1, 0)$ then we get

$$\sum_{i=1}^{2} a_{i1} p_i \geq v_l, \tag{2}$$

and if we take $r = (0, 1)$ then we get

$$\sum_{i=1}^{2} a_{i2} p_i \geq v_l. \tag{3}$$

But by rewriting (1) as

$$r_1 \sum_{i=1}^{2} a_{i1} p_i + r_2 \sum_{i=1}^{2} a_{i2} p_i \geq v_l$$

and using that $r_1 + r_2 = 1, r_1 \geq 0, r_2 \geq 0$, it is clear that (1) holds for all (r_1, r_2) if and only if equations (2) and (3) hold. So now we can rephrase the maximin problem as follows: choose p subject to the constraints $p_1 + p_2 = 1, p_1 \geq 0, p_2 \geq 0$, to maximize the smaller of $\sum_{i=1}^{2} a_{i1} p_i$ and $\sum_{i=1}^{2} a_{i2} p_i$.

This can be posed as the following maximization problem. Find the largest possible value of z, subject to the constaints

$$\sum_{i=1}^{2} a_{i1} p_i \geq z,$$

$$\sum_{i=1}^{2} a_{i2} p_i \geq z,$$

$$p_1 + p_2 = 1, p_1 \geq 0, p_2 \geq 0.$$