Applied Math 226

Computational Assignment 3.

In this computational assignment we consider the same Markov chain approximation as the first assignment. The dynamics are defined by the controlled ${\rm SDE}$

$$dX_1(t) = X_2(t)dt + u(t)dt + \sigma_1 dw_1(t) dX_2(t) = -X_1(t)dt + \sigma_2 dw_2(t),$$

where the control takes values in [-1,1]. The state space is the square $G = (-1,1)^2$. The cost to be minimized is

$$E_x \left[\int_0^\tau \left[c(X(t)) + \frac{b}{2} u(t)^2 \right] dt + g(X(\tau)) \right],$$

where c and g are continuous (do we need c to be positive here?), and τ is the time of first exit from G. The data $\sigma_{1}, \sigma_{2}, b, c$ and g are supplied by the user, but we can assume bounds for each of these.

Write the Matlab code that solves the dynamic programming equation for the approximating chain via iteration in policy space. Allow the user to compare Jacobi and Gauss-Seidel iterations when solving the linear problems for each fixed control, and also allow the user to select either a fixed number of iterations for the iterative solver or a convergence criteria.