Applied Math 226

Computational Assignment 2.

In this computational assignment we will consider a Markov chain approximation for the same two dimensional controlled diffusion as in the last assignment. The dynamics are defined by the controlled SDE

$$\begin{aligned} dX_1(t) &= X_2(t)dt + u(t)dt + \sigma_1 dw_1(t) \\ dX_2(t) &= -X_1(t)dt + \sigma_2 dw_2(t), \end{aligned}$$

where the control takes values in $[-1,1]$. The state space is the square $G=(-1,1)^2$. The cost to be minimized is

$$E_x\left[\int_0^\tau \left[c(X(t))+\frac{b}{2}u(t)^2\right]dt + g(X(\tau))\right],$$

where c and g are continuous (do we need c to be positive here?), and τ is the time of first exit from G. The data σ_1,σ_2, b, c and g are supplied by the user, but we can assume bounds for each of these. Suppose we want to approximate by a Markov chain on the grid $hZ^2 \cap G$, where $1/h$ is an integer and also supplied by the user.

Now we no longer assume the lower bound $\sigma_1 \geq 1$. (Note that in this case we will be able to solve the completely degenerate problem $\sigma_1 = \sigma_2 = 0$.) Construct a locally consistent Markov chain for this problem, and write the Matlab code that implements the Jacobi and Gauss-Seidel versions of the iterative solver analogous to that of the last assignment.