Applied Math 226

Assignment 4.

1. Let

$$\mathcal{L}f(x) = \frac{1}{2}\sum_{i=1}^{n} f_{x_i x_i}(x).$$

Consider a process w on $(\Omega, \mathcal{F}, P)$ that has paths in $C^n[0,\infty)$ and satisfied $w(0) = 0$. Suppose there is a filtration $\mathcal{F}_t$ such that for all $f \in C_0^2(R^n)$,

$$f(w(t)) - f(0) - \int_0^t \mathcal{L}f(w(s)ds$$

is an $\mathcal{F}_t$-martingale. Show that w is $\mathcal{F}_t$-Brownian motion.

2. Consider the control problem

$$V(x) = \inf_u W(x,u)$$

$$W(x,u) = \int_0^\tau k(X(t), u(t))dt$$

$$dX(t) = b(X(t), u(t))dy, X(0) = x,$$

where $\tau = \inf\{t : X(t) \notin G\}$, and the various conditions assumed in class on G, b, k etc. hold. Show that the infimum over ordinary and relaxed controls are the same for this control problem. Given an ordinary control u and $\varepsilon > 0$, show that there is a piecewise constant control u^ε such that $W(x, u^\varepsilon) \leq W(x,u) + \varepsilon$.