

## APPLIED MATH 120

### Review for Exam

As a review for the exam you should make sure that you are comfortable with all the following topics.

- **Deterministic dynamic programming.** How to identify the various components (stages, states, decision variables, dynamics, costs, etc.) for a given problem, and what is the dynamic programming equation for that problem. How to solve (small) dynamic programming problems by hand.
- **Basic terminology and definitions of Markov chains.** What is meant by the transition probability  $p_{ij}$ ? How do we compute  $n$ -step transition probabilities, and what are the relevant linear algebra calculations needed to compute the distribution at time  $n$ , given the distribution at time  $k < n$ ? Should distributions at a given time be written as column vectors or row vectors? You should understand the statement of the Markov and strong Markov properties and be able to tell when they hold and do not hold. Given the description of a problem, you should be able to formulate the related Markov chain by identifying states, transition probabilities, etc.
- **Classification of states.** What are recurrent, absorbing and transient states? What does periodicity mean? What does communication mean? What properties are the same if two states communicate?
- **Invariant (or stationary) distributions.** What are conditions under which a stationary distribution will exist? Are there conditions under which it is unique, How do we compute stationary distributions? You should understand the statement of the ergodic theorem.
- **Other computations.** How to compute more complicated quantities such as mean first passage times, absorption probabilities, and the general problem that includes both as special cases.

I will ask Tom Dean to go over the following problems in recitation. You should try your hand at them first.

1. Problem 10.3-7 in H & L.
2. Consider the transition matrices

$$P_1 = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 1 & 0 & 0 \\ 1/6 & 1/3 & 1/2 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$$

$$P_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

- (a) What are the periods of these chains?
  - (b) Do any of these chains have more than one invariant distribution  $\pi$ ? What is the invariant distribution of  $P_1$ ?
  - (c) Suppose that an initial distribution  $\mu^0$  is given, and that  $\mu^n$  is the distribution at time  $n$ . In which cases will it always be true that  $\mu^n \rightarrow \pi$ ?
  - (d) Compute the mean time till absorption for any initial state for  $P_2$ .
3. Consider a Markov chain with state space  $\{1, \dots, M\}$ . Let  $\tau$  be the first time the chain visits state  $j$ , after first having visited state  $k \neq j$ . Is  $\tau$  a stopping time?