Applied Math 120

Assignment 3, 17 February 2005, due 24 February, 2005.

Do the following problems from H & L.

- pp. 762-763, problems 16.5-2, 16.5-7, 16.5-9.

In addition, do the following.

1. Consider the Markov chain with transition probabilities

$$\begin{pmatrix} 1/6 & 1/2 & 1/3 \\ 0 & 1/4 & 3/4 \\ 2/3 & 1/3 & 0 \end{pmatrix}$$

   Suppose you were given an infinite sequence of iid random variables $U_0, U_1, \ldots$, that are uniformly distributed on $[0,1]$ and an initial condition $i^* \in \{1,2,3\}$. Could you simulate the corresponding Markov chain? In other words, can you build, in terms of i^* and $U_0, U_1, \ldots$, a Markov chain X_n with the given transition probabilities? Hint. Consider partitioning $[0,1]$ into three intervals whose length depends on the current state of the chain.

2. Suppose that the water level behind a dam changes in convenient integer multiples. The dam is height H, a random increase in height A_i occurs each morning, and 1 unit of water is removed each afternoon. The A_i are iid, with $P(A_i = j) = p_j, \ j = 0, 1, \ldots..$ For convenience, let us assume that $p_0 > 0$ and $p_j > 0$ for some $j > 1$. Excess water is spilled (i.e., lost). Is the water level a Markov chain? What can you say about aperiodicity and irreducibility?