Applied Math 120

Assignment 2, 10 February 2005, due 17 February, 2005.

Do the following problems from H & L.

- p. 761, problems 16.4-2, 16.4-5.

In addition, do the following.

1. Let Y_i be the outcome of rolling a fair die, and let N_n be the number of 1's in the first n rolls. Is N_n a Markov chain? If so, what are the transition probabilities?

2. What is the solution to the Gambler's ruin problem when the coin is biased? Suppose that the probability is $0 < q < 1, q \neq 1/2$ that a heads turns up, and let $p = 1 - q$. As usual, we get a dollar for each heads, and lose a dollar for each tails. Compute

$$w_k = P\left(\text{go broke}\,|\text{start at } k\,\right).$$

   To solve the resulting equation, you may want to try solutions of the form $a\rho^k + b$, and figure out what the constants a, b and ρ must be.

3. Let X_n be iid with values in $\{1, ..., K\}$. Let $g(i, j)$ be a function that maps $i \in \{1, ..., K\}$ and $j \in \{1, ..., M\}$ into $\{1, ..., M\}$. Let $Y_0 \in \{1, ..., M\}$ be given and deterministic. Show that

$$Y_{n+1} = g(X_n, Y_n)$$

   always defines a Markov chain. Hint: Look at how Bayes rule was used to show that a sum of iid random variables is Markov. If $q_i = P(X_n = i)$, what are the transition probabilities?