AM 034 — Applied Mathematics - II

Brown University **Fall 2019**
Homework, Set 1 **Due September 18**

Turn this homework in by 4:45pm to the APMA 0340 homework drop-box in the lobby of the Division of Applied Mathematics, 182 George St. You can also hand in your work in your classroom at 12 (noon).

Attach this cover page to the front of your homework and staple all papers together before handing in. (Points will be deducted for failing to do so.)

Show all of your work. Correct answers without work will receive no credit.

> You are encouraged to use either free (*Maxima*, *Sage*, *SymPy*, *R*, Python, and *Octave*) or commercial (*Maple*, *Mathematica*, and MATLAB together with `MuPAD` or Live Editor) software packages.
>
> However, there are restrictions on how you may use software. You may *only* use your software to do the following things: evaluate integrals and simplify complicated algebraic expressions.

Name:

Banner ID:

(Leave this blank)

Problem	Possible	Earned
1	20	
2	20	
3	30	
4	30	
Total	100	

1.1 (20 points) Find a first order system that is equivalent to the second order differential equation

$$y'' - 3\,y' + 4\,y = \cos(2t).$$

1.2 (20 points) Consider a third order constant coefficient differential equation

$$y''' + 3\,y'' - 5\,y' + 7\,y = \cosh(2t).$$

Convert this equation into a system of first order equations and rewrite it in matrix form:

$$\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{f}(t),$$

clearly identifying the 3-by-3 constant matrix $\mathbf{A}$ and 3-column vector $\mathbf{f}$.

1.3 (30 points) Consider the initial value problem

$$\frac{\mathrm{d}y(x)}{\mathrm{d}x} = \sqrt{\frac{1}{4} + y(x)}, \qquad y(0) = \frac{1}{2}.$$

Using Picard's iteration, find first three terms in its approximation

$$y_2(x) = c_0 + c_1 x + c_2 x^2.$$

Convert the given first order differential equation to an equivalent system of equations with polynomial slope functions.

1.4 (30 points) Consider the initial value problem for the simple pendulum equation

$$\frac{\mathrm{d}^2\theta(t)}{\mathrm{d}t^2} + \sin\theta = 0, \qquad \theta(0) = \frac{\pi}{4}, \quad \dot{\theta}(0) = 0.$$

Using Picard's iteration, find first four terms in its approximation

$$\phi_6(x) = c_0 + c_2 t^2 + c_4 t^4 + c_6 t^6.$$

Convert the given second order differential equation to an equivalent system of equations with polynomial slope functions.