APMA 0330 — Applied Mathematics - I

Brown University   **Fall, 2017**
Homework, Set 7   **Due November 29**

7.1 (20 pts) Find the general solution to the following differential equation

$$y'' + 9\, y = 3\, \cot(3x)\,.$$

7.2 (60 pts) In each of problems, express $f(t)$ in terms of the Heaviside function, $H(t)$, and find its Laplace transform.

(a) $f(t) = \begin{cases} 3, & 0 \le t < 3, \\ -2, & 3 \le t < 5, \\ 1, & 5 \le t < 8, \\ 2, & t \ge 8. \end{cases}$

(b) $f(t) = \begin{cases} 0, & 0 \le t < 1, \\ -3, & 1 \le t < 2, \\ 2, & 2 \le t < 3, \\ -4, & 3 \le t < 4, \\ 1, & t \ge 4. \end{cases}$

(c) $f(t) = \begin{cases} 1, & 0 \le t < 3, \\ e^{2(t-3)}, & t \ge 3. \end{cases}$

(d) $f(t) = \begin{cases} t^2, & 0 \le t < 2, \\ t + 1, & t \ge 2. \end{cases}$

(e) $f(t) = \begin{cases} t^2, & 0 \le t < 1, \\ t - 1, & 1 \le t < 2, \\ t^2 + 1, & 2 \le t < 3, \\ 10, & t \ge 3. \end{cases}$

(f) $f(t) = \begin{cases} t - 1, & 0 \le t < 2, \\ 1, & 2 \le t < 4, \\ 5 - t, & 4 \le t < 8, \\ -3, & t \ge 8. \end{cases}$

Hint: The Laplace transform of the power function is

$$\mathcal{L}\left[t^n\right] = \int_0^\infty t^n e^{-\lambda t}\, \mathrm{d}t = \frac{n!}{\lambda^{n+1}}, \qquad n = 0, 1, 2, \ldots.$$

7.3 (20 pts) Find the Laplace transform of the periodic with period $T = 6$ sawtooth function that is the half-wave rectifier of the function

$$f(t) = 1 - t, \quad 0 < t < 3.$$