NAME: _____

APMA 0330 — Applied Mathematics - I

Brown University **Fall, 2017**
Homework, Set 6 **Due November 8**

6.1 (10 pts) Write out the characteristic equation for the given differential equation:

$$\textbf{(a)}\quad y^{(4)} + 5\,y'' - 3y = 0; \qquad\qquad \textbf{(b)}\quad y'' - 7\,y' + 2\,y = 0.$$

6.2 (10 pts) Let α and β be real constants. Consider the differential operator of the second order:

$$L[\mathtt{D}] = (\mathtt{D} - \alpha)^2 + \beta^2,$$

where $\mathtt{D} = \mathrm{d}/\mathrm{d}t$ is the derivative operator. Show that the substitution $y = e^{\alpha t}v(t)$ reduces the differential equation

$$L[\mathtt{D}]y = 0 \qquad \text{or} \qquad \left[(\mathtt{D} - \alpha)^2 + \beta^2\right] y = 0$$

to the canonical equation $\ddot{v} + \beta^2 v = 0$, where dot stands for the derivative with respect to t.

6.3 (10 pts) The Wronskian of two functions is $W(x) = x^2 - 6x + 9$. Are the functions linearly independent or linearly dependent?

6.4 (10 pts) The characteristic equation for a certain differential equation is given. State the order of the differential equation and give the form of the general solution.

$$\textbf{(a)}\quad 2\,\lambda^3 - \lambda^2 - 7\,\lambda + 6 = 0; \qquad\qquad \textbf{(b)}\quad 3\,\lambda^3 - 20\,\lambda^2 + 39\,\lambda = 18.$$

6.5 (20 pts) Find the solution of the given initial value problem.

(a) $6\ddot{y} + \dot{y} - y = 0, \quad y(0) = 1,\ \dot{y}(0) = 2.$
(b) $\ddot{y} - 3\dot{y} = 0, \quad y(0) = 1,\ \dot{y}(0) = 2.$

6.6 (20 pts) Find the form of a particular solution $y_p(t)$ to the following ODEs to be used in the method of undetermined coefficients. **Do not solve for the coefficients!**

(a) $\ddot{y} - 4\,\dot{y} + 4\,y = 3t\,e^{2t},$ (b) $\ddot{y} + 4\,\dot{y} + 13\,y = 4\,e^{-2t}\sin 3t,$
(c) $\ddot{y} + \dot{y} - 2\,y = e^{-2t} + t\,e^{t} + t,$ (d) $\dddot{y} - 5\,\ddot{y} + 7\,\dot{y} - 3\,y = e^{-3t} + t\,e^{t}.$

6.7 [10 pts.] For given family of solutions $c_1x^2 + c_2e^{-3x}\cos 5x$ to a constant coefficient differential equation $L[\mathtt{D}]\,y = 0$, find a linear differential operator of least possible order that annihilates the family.

6.8 [10 pts.] Let $\mathtt{D}$ stand for the derivative operator. Write the general solution of the following differential equation

$$(\mathtt{D} - 2)^3\left[(\mathtt{D} + 5)^2 + 9\right]^2 y = 0.$$