NAME: _____

APMA 0330 — Applied Mathematics - I

Brown University **Fall, 2017**
Homework, Set 5 **Due November 1**

5.1 (30 pts) Consider the initial value problem

$$3\,y' + y = x, \qquad y(0) = -2.$$

Upon introducing uniform grid $x_n = n\,h$ $(n = 0, 1, 2, \ldots)$ with step size $h = 0.1$, find an approximate value of $y(1)$ using

- Euler's rule $y_{n+1} = y_n + h\,f(x_n, y_n)$, and
- Heun's method $y_{n+1} = y_n + \dfrac{h}{2}\,[f(x_n, y_n) + f(x_{n=1}, y_n + h\,f(x_n, y_n))]$.

Compare your answer with the true value $\phi(1)$, where $y = \phi(t)$ is the actual solution.

5.2 (10 pts) Find the separatrix to the differential equation $y' = 4\,\sin(2t) + 2\,y - 8$.

5.3 (20 pts) Solve the equations with the dependent variable missing.

(a) $xy''' - 2y'' = 0$; **(b)** $y''' - y'' = 1$;
(c) $x^2\,y'' + 2y' = 4x$; **(d)** $y'' + y' = 4\,\sinh x$.

5.4 (20 pts) Solve the equations with the independent variable missing.

(a) $y'' + 2y(y')^3 = 0$; **(b)** $2yy'' = y^2 + (y')^2$;
(c) $y\,y'' = (y')^3$; **(d)** $y'' + (y')^2 = 2\,e^{-y}$.

5.5 (10 pts) Determine the longest interval in which the given initial value problem is certain to have a unique twice-differentiable solution.

(a) $(t-1)\,y'' - 4t\,y' + 5\,y = \cos t, \qquad y(0) = 2, \quad y'(0) = 1;$

(b) $t(t^2-4)\,y'' + t\,y' + (\ln|t|)\,y = 0, \qquad y(1) = 0, \quad y'(1) = 2.$

5.6 (10 pts) Find the Wronskian of two solutions of the differential equation $t^2y'' - t(t+2)\,y' + (t+2)\,y = 0$ without solving the equation.