NAME: ____

APMA 0330 — Applied Mathematics - I

Brown University **Fall, 2017**

Homework, Set 4 **Due October 11**

4.1 **(14 pts)** Determine the validity interval for each of the following initial value problems.

(a) $(x^2 - 4)\, y' + x^5 y = x^2 + 1, \quad y(0) = 1;$

(b) $(\cos \pi x)\, y' + (\sin x)\, y = \tan \pi x, \quad y(1) = 1.$

4.2 **(6 pts)** An inductor-resistor series circuit (LR circuit) can be modeled by the following differential equation (the initial condition is assumed to be given $i(0) = i_0 = 0.5$):

$$V_t = V_R(t) + V_L(t) \qquad \Longrightarrow \qquad L\,\frac{\mathrm{d}i}{\mathrm{d}t} + R\,i = V(t) = \begin{cases} 6, & \text{for } 0 < t < 2\,\tau, \\ 0, & \text{otherwise,} \end{cases}$$

where the voltage drop across the resistor is $V_R = i\,R$ (Ohms Law), $R = 1$ being in Ohms, the voltage drop across the inductor is $V_L = L\,\mathrm{d}i/\mathrm{d}t$, $L = 0.1$ being in Henries. The $\tau = L/R$ term in the above equation is known commonly as the time constant. Plot the solution to IVP.

4.3 **(20 pts)** Solve the linear equations. (Parts (c) and (d) do not have a nice integrable function.)

(a) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\sin x + (2y - x)\cos x}{\sin x};$ **(b)** $\dfrac{\mathrm{d}y}{\mathrm{d}x} + y = \dfrac{1}{1 + e^{-x}};$

(c) $\dfrac{\mathrm{d}y}{\mathrm{d}x} + y\,\sin x = 2x\,\sin x;$ **(d)** $\dfrac{\mathrm{d}y}{\mathrm{d}x} - y\,\ln x = x^2.$

4.4 **(20 pts)** Find the particular solution to the given initial value problem.

(a) $x\,y' + (x+2)\,y = 2\,\sin x, \qquad y(\pi) = -1;$

(b) $x^2 y' - 4x\,y = x^4, \qquad y(1) = 2;$

(c) $y' + 3y = f(x) = \begin{cases} 9x, & \text{if } 0 \le x < 1, \\ 9, & \text{if } 1 \le x < \infty; \end{cases}, \qquad y(0) = 0;$

(d) $x^2 y' + 2x\,y = \cos x, \qquad y(\pi) = 0.$

4.5 **(20 pts)** Solve the following Bernoulli equations.

(a) $x\,y' - y = -3\,x^4 y^3;$ **(b)** $x\,y' = (x+1)\,y - 2\,y^3;$

(c) $3y' + 2y^4 x\,e^{-3x} = y;$ **(d)** $y' + 2y\,\csc(2x) = y^2.$

4.6 **(20 pts)** Solve the initial value problems for the Bernoulli equation.

(a) $x\,y' + y = x^4 y^3, \qquad y(1) = 1/4;$ **(b)** $x\,y' + 3\,y = x^3 y^2, \qquad y(1) = 1/2.$