NAME: ____

APMA 0330 — Applied Mathematics - I

Brown University **Fall, 2017**
Homework, Set 3 **Due October 4**

3.1 (12 pts) Given a potential function $\psi(x,y)$, find the exact differential equation $\mathrm{d}\psi(x,y) = 0$.

(a) $\psi(x,y) = 3\,x^2 + 5\,y^2$; **(b)** $\psi(x,y) = \exp(3\,x^2 y^3)$;
(c) $\psi(x,y) = \ln(x^3 y^4)$; **(d)** $\psi(x,y) = (2x+3y-5)^2$.

3.2 (20 pts) Show that the following differential equations are exact and solve them

(a) $3x^2y^2\,y' + 2y^3 x = 0$; **(b)** $y\,\left(e^{xy}+y\right)\,\mathrm{d}x + x\,\left(e^{xy}+2y\right)\,\mathrm{d}y = 0$;
(c) $(3x^2y + 2x\,e^y)\,\mathrm{d}x + (x^2 e^y + x^3)\,\mathrm{d}y = 0$; **(d)** $(2xy^2-3)\,\mathrm{d}x + (2x^2y+y^2)\,\mathrm{d}y = 0$.

3.3 (24 pts) Are the following equations exact? Solve the initial value problems.

(a) $\sin\pi x\,\cos 3\pi y\,\mathrm{d}x + 3\cos\pi x\,\sin 3\pi y\,\mathrm{d}y$, $y(3/2) = 1/3$;

(b) $6xy\,\mathrm{d}x + (3x^2+4y^3)\,\mathrm{d}y = 0$, $y(0) = 4$;

(c) $(3x^2y-5)\,\mathrm{d}x + (x^3+6y^2)\,\mathrm{d}y = 0$, $y(1) = 2$;

(d) $(\cos\theta - 2r\cos^2\theta)\,\mathrm{d}r + r\sin\theta(2r\cos\theta - 1)\,\mathrm{d}\theta = 0$, $\quad r(\pi/4) = 1$.

3.4 (24 pts) Show that the given equations are not exact, but become exact when multiplied by the corresponding integrating factor. Find an integrating factor as a function of x only and determine a potential function for the given differential equations.

(a) $y' + y(1+2x) = 0$; **(b)** $x^3\,y' = x^2 y + 3x$;
(c) $(yx^3e^{xy} - 2y^3)\,\mathrm{d}x + (x^4e^{xy} + 3xy^2)\,\mathrm{d}y = 0$; **(d)** $4\,\mathrm{d}x - e^{y-2x}\,\mathrm{d}y = 0$.

3.5 (20 pts) Find an integrating factor as a function of y only and determine the general solution for the given differential equations.

(a) $(y+3)\,\mathrm{d}x - (x-y)\,\mathrm{d}y = 0$; **(b)** $\left(\frac{y}{x} - 1\right)\,\mathrm{d}x + \left(2y^2 + 1 + \frac{x}{y}\right)\,\mathrm{d}y = 0$;
(c) $(2xy^2+3y)\,\mathrm{d}x - 3x\,\mathrm{d}y = 0$; **(d)** $y(x+y+2)\,\mathrm{d}x + x(x+3y+4)\,\mathrm{d}y = 0$.