NAME: ____

APMA 0330 — Applied Mathematics - I

Brown University **Fall, 2017**

Homework, Set 1 **Due September 20**

1.1 Consider a population of field mice who inhabit a certain rural area. Suppose that several owls live in the same neighborhood and they kill 12 mice per day. Then the mouse population $p(t)$, t is measured in months, satisfies the differential equation

$$\dot{p} \equiv \mathrm{d}p/\mathrm{d}t = p/3 - 360.$$

(a) Plot a direction field for the given differential equation using one of your lovely software package. Provide the codes of your plot or state what resources did you use. Based on the slope field, determine the behavior of $p(t)$ as $t \to \infty$.

(b) Determine an equilibrium solution.

(c) Find the time at which the population becomes extinct if $p(0) = 900$.

(d) Find the initial population $p(0)$ if the population is to become extinct in 2 years.

1.2 Suppose that the population of fish is modeled by the logistic equation:

$$\dot{P}(t) = 0.1\, P\, (1 - P/500)\,.$$

(a) Find critical points and identify them as unstable of asymptotically stable.

(b) Find the solution of the above logistic equation subject to the initial condition $P(0) = 250$.

1.3 Consider the initial value problem

$$y' = x - y, \qquad y(0) = 1.$$

(a) Convert the given initial value problem to the integral equation:

$$y(x) = y_0 + \int_{x_0}^{x} f\left(s, y(s)\right) \mathrm{d}s,$$

by identifying x_0, y_0, and the slope function $f(x, y)$.

(b) Use Picard's iterations

$$\phi_{n+1}(x) = y_0 + \int_{0}^{x} f\left(s, \phi_n(s)\right) \mathrm{d}s$$

to find the unique solution of the given initial value problem. *Hint:* You may want to use the Maclauren series for exponential function: $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$.

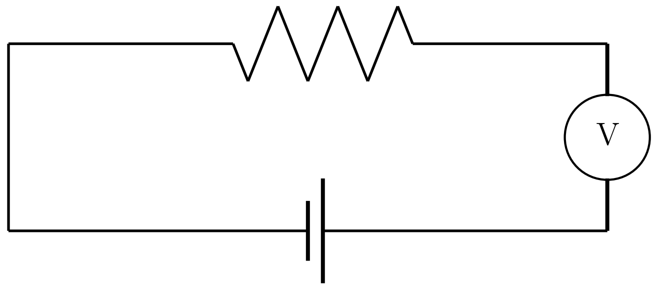

Figure 1: The electric circuit of Problem 1.5.

1.4 A certain drug is being administrated intravenously to a hospital patient. Fluid containing $6\,\mathtt{mg/cm}^3$ of the drug enters the patient's bloodstream at a rate of $120\,\mathtt{cm}^3\mathtt{/h}$. The drug is absorbed by body tissues or otherwise leaves the bloodstream at a rate proportional to the amount present, with a rate of $0.5\,\mathtt{(h)}^{-1}$.

(a) Assuming that the drug is always uniformly distributed throughout the bloodstream, write a differential equation for the amount of the drug that is present in the bloodstream at any time.

(b) How much of the drug is present in the bloodstream after a long time?

1.5 Consider an electrical circuit containing a capacitor, resistor, and battery. The charge $Q(t)$ on the capacitor satisfies the equation

$$R\,\frac{\mathrm{d}Q}{\mathrm{d}t} + \frac{Q}{C} = V,$$

where R is the (constant) resistance, C is the (constant) capacitance, and V is the constant voltage supplied by the battery.

(a) If $Q(0) = 0$, find $Q(t)$ at any time, and sketch the graph of Q versus t.

(b) Find the limiting value Q_L that $Q(t)$ approaches after a long time.

(c) Suppose that $Q(t_1) = Q_L$ and that at time $t = t_1$ the battery is removed and the circuit is closed again. Find $Q(t)$ for $t > t_1$ and sketch its graph.