Formulas

Euler's formulas:

$$e^{\mathbf{j}\theta} = \cos\theta + \mathbf{j}\sin\theta, \qquad \sin\theta = \frac{1}{2\mathbf{j}}\,e^{\mathbf{j}\theta} - \frac{1}{2\mathbf{j}}\,e^{-\mathbf{j}\theta}, \qquad \cos\theta = \frac{1}{2}\,e^{\mathbf{j}\theta} + \frac{1}{2}\,e^{-\mathbf{j}\theta}.$$

Laplace Transformation: For $f(t)$, defined for $t \geq 0$,

$$\mathcal{L}[f(t)](\lambda) \equiv f^L(\lambda) = \int_0^\infty e^{-\lambda t} f(t)\,\mathrm{d}t$$

is said to be the *Laplace transform* of f, if the integral converges for some value $\lambda = \lambda_0$.

Linearity:

$$\mathcal{L}[c_1 f_1 + c_2 f_2] = c_1\mathcal{L}[f_1] + c_2\mathcal{L}[f_2]$$

Convolution: The convolution of two functions f and g, defined on the half-line $[0,\infty)$, is the integral

$$(f * g)(t) = \int_0^t f(t-\tau)g(\tau)\,\mathrm{d}\tau = (g * f)(t).$$

The **Heaviside function:** $H(t)$ is defined to be

$$H(t) = \begin{cases} 1, & t > 0 \\ 1/2, & t = 0 \\ 0, & t < 0 \end{cases}$$

#	$f(t)$	$\mathcal{L}[f(t)](\lambda)$
1.	$H(t)$	$\frac{1}{\lambda}$
2.	$H(t-a)$	$\frac{1}{\lambda}\, e^{-a\lambda}$
3.	t	$\frac{1}{\lambda^2}$
4.	$t^n,\ n=1,2,\ldots$	$\dfrac{n!}{\lambda^{n+1}}$
5.	$e^{\alpha t}$	$\frac{1}{\lambda-\alpha},\quad \lambda>\alpha$
6.	$\sin\alpha t$	$\frac{\alpha}{\lambda^2+\alpha^2},\quad \Re\lambda>0$
7.	$\cos\alpha t$	$\frac{\lambda}{\lambda^2+\alpha^2},\quad \Re\lambda>0$

Properties of the Laplace transform

1° **The differential rule:**

$$\mathcal{L}\left[f'(t)\right](\lambda)=\lambda\,\mathcal{L}[f](\lambda)-f(0) \qquad \mathcal{L}\left[f''(t)\right](\lambda)=\lambda^2\,\mathcal{L}[f](\lambda)-\lambda f(0)-f'(0).$$

2° **The convolution rule:**

$$\mathcal{L}[f*g](\lambda)=\mathcal{L}[f](\lambda)\mathcal{L}[g](\lambda)$$

3° **The shift rules:**

$$\mathcal{L}\left[H(t-a)g(t-a)\right](\lambda)=e^{-a\lambda}\,\mathcal{L}[g](\lambda),\quad a>0,$$

and

$$\mathcal{L}\left[H(t-a)g(t)\right](\lambda)=e^{-a\lambda}\,\mathcal{L}[g(t+a)](\lambda),\quad a>0,$$

where H is the Heaviside function.

4° **The attenuation rule:**

$$\mathcal{L}\left[e^{at}\,f(t)\right](\lambda)=\mathcal{L}[f](\lambda-a)$$

5° **The Laplace transform of periodic functions**

If $f(t)=f(t+\omega)$, then

$$f^L(\lambda)=\frac{1}{1-e^{-\omega\lambda}}\int_0^\omega e^{-\lambda t}\,f(t)\,\mathrm{d}t.$$