Center for Fluid Mechanics, Division of Applied Mathematics
Fluids and Thermal Systems, School of Engineering
Joint Seminar Series

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3:00pm
Barus & Holley, Room 190

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Fractional Calculus: A Tool to Model Complex Dynamics in Multi-scale Structures

Fractional order models provide a heuristic approach to the description of complex circuits and systems. Instead of simply extending the structure, composition or number of components in a linear system, the 'fractional' approach is to generalize the order of the integer derivatives that describe key dynamic processes (e.g., battery charging, viscoelastic creep, or electric/magnetic polarization). As noted by the mathematician, M. Kac, (Some Mathematical Models in Science, 7 Nov. 1969, Vol. 166 pp. 695-699, *Science*), "success is characterized by the fidelity with which such models fit the observed phenomena, and by the sharpness of the questions they pose about the underlying physics." Fractional order models are sometimes criticized for what they are not – conceptual models of fundamental laws or theories. Therefore, it is important to identify the properties of a complex system that suggest it is appropriate for fractional order generalization. Just as Brownian motion in a stochastic system follows from assumptions that the component particles are identical and independent of each other, fractional order models appear to be most useful for systems that relax these assumptions, and exhibit some degree of memory or nonlocal behavior.

The theme of this presentation is that fractional (non-integer order) calculus can provide a basis for a greater understanding of the molecular events that occur in biological tissues. Such an understanding is fundamental in bioengineering where engineers seek to describe the underlying multi-scale processes that occur, for example, when tissues are electrically stimulated or mechanically stressed. Fractional order models work well in physics, chemistry and rheology, particularly in describing dielectrics and viscoelastic materials over extended ranges of time and frequency. Further, in heat transfer and electrochemistry, the half-order fractional integral is a natural convolution operator connecting the applied gradients (thermal or material) with the diffusion of ions or heat. Can fractional calculus uncover similar relatively simple links between stress and strain in load-bearing tissues, the electrical impedance of implanted cardiac pacemaker electrodes, or in predicting changes in the shear modulus of tumors developing in breast tissue? Since the constitutive properties of tissue depend on the composition and micro-scale architecture of the cellular and extracellular networks, the challenge is to develop non-invasive modeling, visualization and assessment tools that predict macro-scale mechanical performance from micro-scale observations or measurements. In this seminar I will describe some of the characteristics of fractional calculus that make it well suited for modeling engineered systems, and outline three areas of bioengineering research where fractional calculus is being applied.