

Conjugate Gradient Method: Toeplitz Matrices

Consider five symmetric Toeplitz matrices $\mathbf{A}$ with entries given by ($k = 1, \dots, n$)

$$a_k^{(1)} = 1/k, \quad a_k^{(2)} = 1/\sqrt{k}, \quad a_k^{(3)} = 1/k^2, \quad a_k^{(4)} = k, \quad a_k^{(5)} = \cos k/k$$

and an arbitrary nonzero vector $\mathbf{b}$. Use PCGM to solve the systems $\mathbf{Ax} = \mathbf{b}$ with circulant preconditioners and experiment with different types (i.e., number of Toeplitz diagonals employed). Compare your results without CG preconditioning in terms of the number of iterations for sizes up to $n = 100$ and tolerance levels just above single machine accuracy.