1. **Convection terms in two dimensions.** A convection term can be added to the two-dimensional model problem in the form

\[-\epsilon (u_{xx} + u_{yy}) + au_x = f(x).\]

Using a 2D equidistant grid and second-order central finite difference approximations, find the system of linear equations associated with this problem. What condition must be met by \(a\) and \(\epsilon\) for the associated matrix to be diagonally dominant?

2. **Gauss-Seidel eigenvalues and eigenvectors.**

   (a) Show that the eigenvalue problem for the Gauss-Seidel interaction matrix, \(R_G w = \lambda w\), may be expressed in the form \(Uw = (D - L)\lambda Iw\), where \(A = D - L - U\).

   (b) Write out the equations of this system and note the boundary condition \(w_0 = w_n = 0\). Look for solutions of this system of equations of the form \(w_j = \mu^j\), where \(\mu \in \mathbb{C}\) must be determined. Show that the boundary conditions can be satisfied only if \(\lambda = \lambda_k = \cos^2\left(\frac{k\pi}{n}\right), 1 \leq k \leq n - 1\).

   (c) Show that the eigenvector associated with \(\lambda_k\) is \(w_{k,j} = \lambda_k^{j/2} \sin\left(\frac{jk\pi}{n}\right)\).

3. **Effect of full weighting.** Verify that \(I_h w_k^h = \cos^2\left(\frac{k\pi}{2n}\right) w_k^h\), where \(w_{k,j}^h = \sin\left(\frac{jk\pi}{n}\right), 1 \leq k < \frac{n}{2}\), and \(I_h^2\) is the full weighting operator.

4. **Effect of full weighting.** Verify that \(I_h w_{k'}^h = -\sin^2\left(\frac{k\pi}{2n}\right) w_k^h\), where \(k' = n - k, 1 \leq k < \frac{n}{2}\), and \(I_h^2\) is the full weighting operator. What happens to \(w_{n/2}^h\) under full weighting?

5. **Complementary modes.** Show that the complementary modes \(\{w_k^h, w_{k'}^h\}\) on \(\Omega^h\) are related by \(w_{k',j}^h = (-1)^{j+1} w_{k,j}^h\), where \(k' = n - k\).