Homework 2

Problem 1 – Classical vs. Modified Gram-Schmidt

Construct a square matrix $A = U * S * V$ where U and V are random orthogonal matrices and S is a diagonal matrix with exponentially graded entries from 2^{-1} to 2^{-80}.

Use the classical and modified Gram-Schmidt algorithms to obtain QR factorizations of A. Do you obtain the same answers for both algorithms? Also, plot r_{jj} versus j for both cases. What do you observe?

Problem 2 – Householder transformation

Prove that $Hx = -sign(x_1)||x||_2 e_1$, for a nonzero vector $x \neq e_1$, where H is the Householder matrix.

Problem 3 - QR and LU decompositions

Consider the $n \times n$ matrix A with entries 1's in the main diagonal and in the last column, -1's below the main diagonal, and 0's everywhere else. Perform QR and LU factorizations of this matrix for various values of $n = 10, 100, 1000, 10000$, etc and plot the growth factor as a function of n. What do you observe?

Here we define the growth factor ρ as $\rho_{LU} = \frac{\max_{i,j} |U_{ij}|}{\max_{i,j} |a_{ij}|}$ for the LU decomposition and $\rho_{QR} = \frac{\max_{i,j} |r_{ij}|}{\max_{i,j} |a_{ij}|}$ for the QR decomposition.

Problem 4 – LU decomposition

Consider the $n \times n$ matrix A in the linear system $Ax = b$. The (semi)-bandwidth of A is m.

Estimate the computational work in terms of n, m in performing the LU decomposition of A and also the work for the forward and backward solves. Please justify your answers.