Computational Fluid Dynamics - AM 258

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Homework #1

1. Show that the viscous Burger's equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2}$$

   is parabolic in x, t.

   Hint: Introduce $v = \dfrac{\partial u}{\partial x}$ as a second variable, and study the resulting system.

2. Consider the 1D Euler equations:

$$\begin{aligned} \frac{\partial \rho}{\partial t} &+ u\frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} = 0 \\ \frac{\partial u}{\partial t} &+ u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0 \\ \frac{\partial H}{\partial t} &+ u\frac{\partial H}{\partial x} = \frac{1}{\rho}\frac{Dp}{Dt} \end{aligned}$$

   where the total enthalpy $H = E + \dfrac{p}{\rho}$ and D/Dt denotes total derivative.

   Introduce the isentropic assumption

$$\frac{\partial p}{\partial \rho} = c^2 \quad (c = \text{speed of sound})$$

   and replace the third equation by:

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + \rho c^2 \frac{\partial u}{\partial x} = 0$$

   Show that the resulting system is hyperbolic and find the corresponding eigenvalues.

   Answer: $\lambda^{(1)} = u$; $\lambda^{(2)} = u + c$; $\lambda^{(3)} = u - c$